The Qualitative Trajectory Calculus on Networks

Peter Bogaert[1], Nico Van de Weghe[1], Anthony G. Cohn[2], Frank Witlox[1], Philippe De Maeyer[1]

[1]Ghent University, Department of Geography, Krijgslaan 281, 9000 Gent, Belgium
{Peter.Bogaert, Nico.VandeWeghe, Frank.Witlox, Philippe.DeMaeyer}@UGent.be

[2] University of Leeds, School of Computing, Leeds LS2 9JT, United Kingdom
A.G.Cohn@leeds.ac.uk.

Abstract. Moving objects are commonly handled using quantitative methods and information. However, in many cases, qualitative information can be more efficient and more meaningful than quantitative information. A lot of research has been done in generating, indexing, modelling and querying network-based moving objects, but little work has been done in building a calculus of relations between these objects in a qualitative way. In this paper, we introduce a formal definition of how to represent and reason about the relative trajectories of pairs of objects moving along a network.

1 Introduction

In the literature there are two standard approaches when dealing with topological relations between two regions. From the viewpoint of databases, Egenhofer et al. [6] worked out the 9-Intersection Model. Independently, Randell et al. [14] studied the subject from an artificial intelligence point of view resulting in the Region Connection Calculus (RCC). Both approaches reach the same conclusion: a set of eight jointly exhaustive and pairwise disjoint (JEPD) topological relations between two regions without holes. Assuming continuous motion, there are constraints upon the ways these relations change. For example, two objects cannot change their relation from disjoint (DC) to partial overlap (PO) without touching (EC) each other first. These possible changes can graphically be represented by means of a Conceptual Neighbourhood Diagram (CND). CND^s have been introduced in the temporal domain [7], and have been widely used in spatial reasoning, e.g.: for topological relations [14,,4]; cardinal directions [5], and for relative orientation [8]. CND^s are typically used for qualitative simulation to predict what will happen in the future. Two relations between entities are conceptual neighbours, if they can be transformed into one another by continuously deforming, without passing another qualitative relation; a CND describes all the possible transitions between relations that can occur [7].

In the real world, most moving objects have a disjoint (DC) relation. A potential problem here is that both the RCC calculus and the 9-Intesection Model can not further differentiate between disjoint objects, nor indeed could any purely topological representation. Moreover, when dealing with moving point objects (MPO's), there are, according to the 9-Intersection Model, only two topological relations between points (i.e. disjoint and meet). Hence these approaches fail to make explicit the level of disjointness of how two or more objects move with respect to each other.. Obvious examples where this type of information is of vital importance is the case of two airplanes and to know whether they are likely to stay in a disjoint relation, if not the consequences can be catastrophic. Therefore, Van de Weghe [16] introduced the Qualitative Trajectory Calculus (QTC). This calculus deals with qualitative relations between two disjoint, moving, point-like objects. Here we want to focus on how QTC can be of use when dealing with moving object in a network situation.

The structure of this paper is as follows. Section 2 gives the definition of QTC which is the basis for the Qualitative Trajectory Calculus on Networks (QTC_N) and describes the usefulness of qualitative relations. Section 3 defines QTC_N and gives an overview of all possible relations and transitions between these relations. Final conclusions and directions for future work are given in section 4.

2 The Qualitative Trajectory Calculus

2.1 Qualitative Relations

Reasoning can be performed with quantitative as well as qualitative information. Typically, when working with quantitative information, a predefined unit of a quantity is used [12]. For example, one could say that a car drives at 30 km/h. In the qualitative approach, continuous information is being quantised or qualitatively discretised by landmarks separating neighbouring open intervals, resulting in discrete quantity spaces [22]. Qualitative reasoning only studies the essence of information, represented as a small set of symbols such as the quantity space {–, 0, +} consisting of the landmark value '0' and its neighbouring open intervals '–' and '+'. For example, if one does not know the precise speed of a car and a bicycle, but knows that the speed of the car is higher than the speed of the bicycle, one can label this with the qualitative value '+', meaning that the car is moving faster than the bicycle. One could also say that the bicycle is moving slower than the car, by giving the qualitative value '−' to this relation. Finally, both objects can also move at the same speed, resulting in a qualitative value '0'. One thing is for sure; the speed of a car cannot change from being higher than the speed of the bicycle to being lower than the speed of the bicycle, without passing the qualitative value '0'.

There are a variety of reasons why qualitative reasoning claims their place next to, or complementary to, quantitative reasoning in areas such as Artificial Intelligence and Geographic Information Science. First of all, qualitative knowledge tends to be less expensive than its quantitative counterpart, since it contains less information [8]. Moreover, qualitative data often provide, at an early stage of research, an ideal way to deliver insights in order to identify quickly potential problems that warrant more detailed quantitative analysis [13]. In addition, humans usually prefer to communicate in qualitative categories, supporting their intuition, than using quantitative measures.

2.2 Formal Definition

The Qualitative Trajectory Calculus (QTC) examines changes in qualitative relations between two disjoint, point-like objects. Depending on the level of detail and the number of spatial dimensions, different types of QTC are defined in [16], all belonging to QTC-Basic (QTC_B) [18, 20] or QTC-Double-Cross (QTC_C) [19, 21]. In this section, we focus on QTC-Basic, since this is the basis for defining the Qualitative Trajectory Calculus on Networks (QTC_N).

QTC_B is developed for moving objects in one (QTC_{B1}) or two dimensions (QTC_{B2}). In QTC_{B1}, it is assumed that the movement of two objects is restricted to a line (e.g. two trains moving on a railroad track). In QTC_{B2}, two objects can move freely in a plane (e.g. two ships floating on an ocean). The landmark to describe the qualitative relations in QTC_B is the distance at time *t* between the two objects. A typical, essential characteristic in both cases of QTC_B is the three character label representing the qualitative movement between two objects. This label represents the following three relationships:

Assume two objects[1] *k* and *l*

1. Movement of the first object *k*, with respect to the position of the second object *l* at time point *t*:

   −: *k* is moving towards *l*:

$$\begin{aligned}&\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) > \mathrm{d}(k \mid t, l \mid t))) \wedge \\ &\qquad \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) > \mathrm{d}(k \mid t^+, l \mid t)))\end{aligned} \tag{1}$$

   +: *k* is moving away from *l*:

$$\begin{aligned}&\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) < \mathrm{d}(k \mid t, l \mid t))) \wedge \\ &\qquad \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) < \mathrm{d}(k \mid t^+, l \mid t)))\end{aligned} \tag{2}$$

   0: *k* is stable with respect to *l* (all other cases):

$$\begin{aligned}&\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) = \mathrm{d}(k \mid t, l \mid t))) \wedge \\ &\qquad \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) = \mathrm{d}(k \mid t^+, l \mid t)))\end{aligned} \tag{3}$$

$$\begin{aligned}&\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) = \mathrm{d}(k \mid t, l \mid t))) \wedge \\ &\qquad \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) < \mathrm{d}(k \mid t^+, l \mid t)))\end{aligned} \tag{4}$$

$$\begin{aligned}&\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) = \mathrm{d}(k \mid t, l \mid t))) \wedge \\ &\qquad \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) > \mathrm{d}(k \mid t^+, l \mid t)))\end{aligned} \tag{5}$$

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[1] We introduce the following notation for QTC:
$x|t$ denotes the position of an object *x* at time *t*;
$\mathrm{d}(u,v)$ denotes the distance between two positions *u* and *v*;
$v_x|t$ denotes the speed of *x* at time *t*;
$t_1 < t_2$ denotes that t_1 is temporally before t_2;
t^- denotes the time period immediately before *t*;
t^+ denotes the time period immediately after *t*.

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) > \mathrm{d}(k \mid t, l \mid t))) \wedge \\ \exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) = \mathrm{d}(k \mid t^+, l \mid t))) \tag{6}$$

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) > \mathrm{d}(k \mid t, l \mid t))) \wedge \\ \exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) < \mathrm{d}(k \mid t^+, l \mid t))) \tag{7}$$

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) < \mathrm{d}(k \mid t, l \mid t))) \wedge \\ \exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) = \mathrm{d}(k \mid t^+, l \mid t))) \tag{8}$$

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) < \mathrm{d}(k \mid t, l \mid t))) \wedge \\ \exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) > \mathrm{d}(k \mid t^+, l \mid t))) \tag{9}$$

2. The movement of the second object *l*, with respect to the position of the first object *k* at time point *t* can be described as in 1. with *k* and *l* interchanged, and hence:
   −: *l* is moving towards *k*
   +: *l* is moving away from *k*
   0: *l* is stable with respect to *k*
3. Relative speed of the first object *k* at time point *t*, with respect to the second object *l* at time point *t*:
   −: *k* is slower than *l*:

$$v_k \mid t < v_l \mid t \tag{10}$$

   +: *k* is faster than *l*:

$$v_k \mid t > v_l \mid t \tag{11}$$

   0: *k* and *l* are equally fast:

$$v_k \mid t = v_l \mid t \tag{12}$$

Note that with the introduced three characters, relationships between the two objects can now be described. For example, if object *k* and object *l* are both moving towards each other (resulting in a '−' for the first and a '−' for the second character) and object *k* is moving slower than object *l* (resulting in a '−' for the third character), this will result in a $(- - -)_{\mathrm{B}}$ label.

By definition, in QTC$_{\mathrm{B}}$, there are theoretically 3^3 (27) different relationships. However, in QTC$_{\mathrm{B1}}$ only 17 real-life (in theory feasible) possibilities remain. For example, it is impossible for one object to be faster than the other if both objects are not moving (Figure 1: 5a or 5c). Note that in QTC$_{\mathrm{B2}}$, all 27 relations are possible (Figure 2).

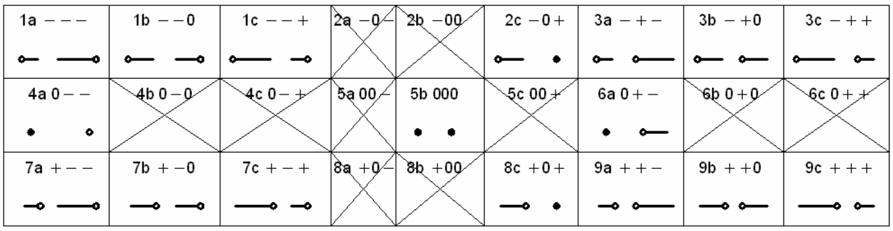

Fig. 1. 17 real-life QTC$_{B1}$ labels[2]

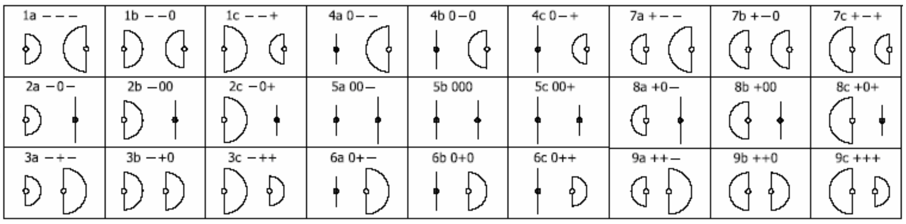

Fig. 2. 27 real-life QTC$_{B2}$ labels[3]

2.3 Theory of Dominance

Adopting the previously defined concept of conceptual neighbours [7] to, '*two trajectory pairs are conceptual neighbours if they can directly follow each other during a continuous movement'*, gives the possibility to create a Conceptual Neighbourhood Diagram (CND) for QTC$_B$.

The construction of the CND is based on the concept of 'dominance space', introduced by Galton [9]. Galton outlined the temporal nature of transitions between qualitative variables, defining some important restrictions concerning the dominance between binary qualitative relations. Central in his theory of dominance are the constraints imposed by continuity. Consider the qualitative distinction between '–', '0' and '+', then a variable capable of assuming any of these three descriptions may

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[2] The left and right dot represent respectively the positions of *k* and *l*. A dot is filled if the object can be stationary. The line segments represent the potential object movements. Note that the lines can have different lengths giving the difference in relative speed. The line segments represent whether each object is moving towards or away from the other.

[3] The icons contain line segments with the point object positioned in the middle. The line segment denotes the possibility to move to both sides of the point object. The filled dot represents the case when the object can be stationary. An open dot means that the object cannot be stationary. The icons also contain crescents with the point object in the middle of its straight border. The crescent denotes an open polygon. If a crescent is used, then the movement starts in the dot and ends somewhere on the curved side of the crescent.

change between them. However, a direct change from '–' to '+' and vice versa is impossible, since such a change must always pass the qualitative value '0'. This landmark value '0' only needs to hold for an instant. On the other hand, the '+' of a variable, when changing from '0' to '+' and back to '0', must hold over an interval [10]. Let us briefly illustrate this point. The issue is that between any two points of a continuous trajectory we can always find, or at least imagine, another intermediate point. In order words, applied to a real number line, between zero and any positive real number, one can always find another positive real number: $0 < 10 < 100$; $0 < 1 < 10$; $0 < 0.1 < 1$; $0 < 0.01 < 0.1$; $0 < 0.001 < 0.01$, etc. It then follows that it is impossible that + only holds over an instant of time. Dual reasoning applies for the change from '0' to '–'. In Galton's [10] terms, we state that '0' dominates '–' and '+', and that '–' and '+' are dominated by '0'.

Based on the concept of dominance, one can construct a *dominance space*. This is a space containing qualitative values with their dominance relations. Figure 3 represents a basic example of the dominance space between the qualitative values '–', '0' and '+', where '–' denotes the set of negative real numbers, '0' is the landmark value, and '+' denotes the set of positive real numbers. It follows from Figure 3 that:

- there is a connection between '–' and '0', and the arrow is in the direction of '–', thus, a transition from '–' to '0' can occur and vice versa, with '0' dominating '–';
- there is a connection between '0' and '+', and the arrow is in the direction of '+', thus, a transition from '0' to '+' can occur and vice versa, with '0' dominating '+';
- there is no direct connection between '–' and '+', thus, a transition from '–' to '+' and vice versa can only occur by passing through '0'.

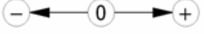

Fig. 3. Dominance space in one dimension

A set of dominance spaces can be combined in order to build composite dominance spaces [10]. In Figure 4a, two *one-dominance*[4] spaces are visualised. A combination of these one-dominance spaces leads to a two-dominance space shown in 4b. The

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[4] A one-dominance space is a dominance space where all conceptual distances are equal to one. In order to explain the concept of conceptual difference, let us consider three examples:

1. Assume that R_1 and R_2 only differ in one character that can change continuously between both states without passing through an intermediate qualitative value. Then the conceptual distance between R_1 and R_2 is one. Example: $R1 = (000)_{B1}$ and $R_2 = (0{+}0)_{B1}$ → conceptual distance is one.
2. Assume that R_1 and R_2 only differ in one character that cannot change continuously between both states without passing through an intermediate qualitative value. Then the conceptual distance between R_1 and R_2 is composed of sub-distances. Example: Suppose $R_1 = (0{-}0)_{B1}$ and $R_2 = (0{+}0)_{B1}$. Then the conceptual distance between $(0{-}0)_{B1}$ and $(000)_{B1}$ is one, and the conceptual distance between $(000)_{B1}$ and $(0{+}0)_{B1}$ is one. Thus, the conceptual distance between $(0{-}0)_{B1}$ and $(0{+}0)_{B1}$ is two.
3. Assume that R_1 and R_2 differ in multiple characters. Then the conceptual distance is the sum the sub-distances determined for each individual character (i.e. the Manhattan distance). Example: Suppose $R_1 = (-\,-\,0)_{B1}$ and $R_2 = (+\,+\,0)_{B1}$. Then the conceptual distance for the first character is two, and the conceptual distance for the second character is two. Thus, the conceptual distance between $R_1 = (-\,-\,0)_{B1}$ and $R_2 = (+\,+\,0)_{B1}$ is four.

disjunction of the two one-dominance spaces and the two-dominance space leads to an *overall dominance* space in two dimensions represented in Figure 4c.

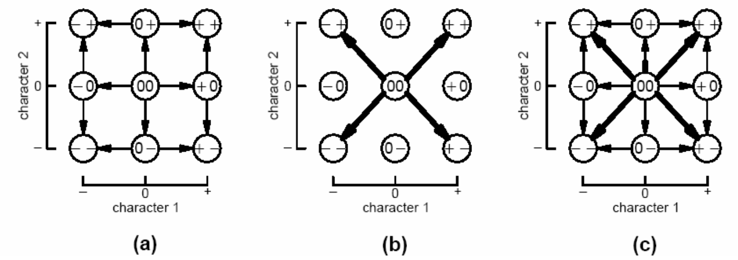


Fig. 4. Combination of dominance spaces

2.4 Conceptual Neighbourhood Diagrams for QTC_B

Since QTC_B describes three orthogonal qualitative values we can use the theory of dominance to construct the overall dominance space for three dimensions. Figure 5 shows the one-dominance space for each dimension.

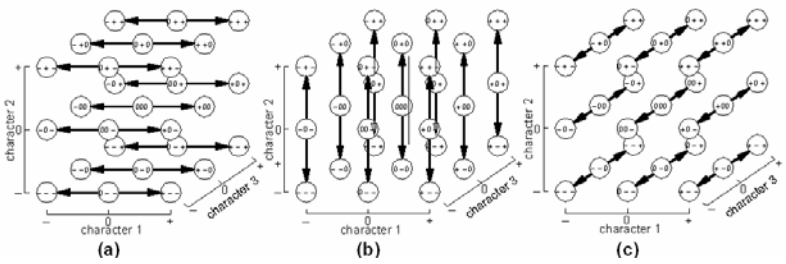
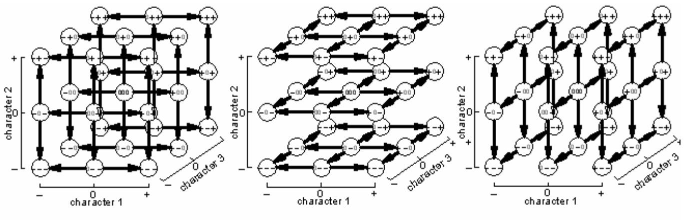


Fig. 5. One-dominance spaces for three dimensions

These three one-dominance spaces can be combined pair wise (Figure 6a), leading to three composite two-dominance spaces as shown in Figure 6b.



(a)

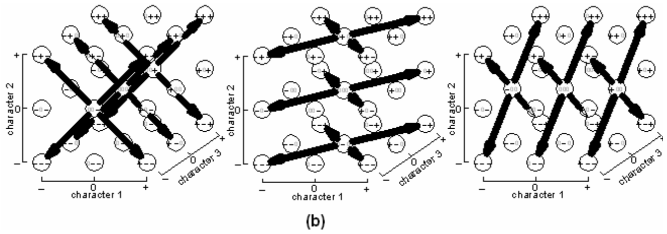


Fig. 6. Construction of two-dominance spaces

Furthermore, the two-dominance spaces can each be combined with their orthogonal one-dominance space (Figure 7a). These combinations all result in the same three-dominance space (Figure 7b).

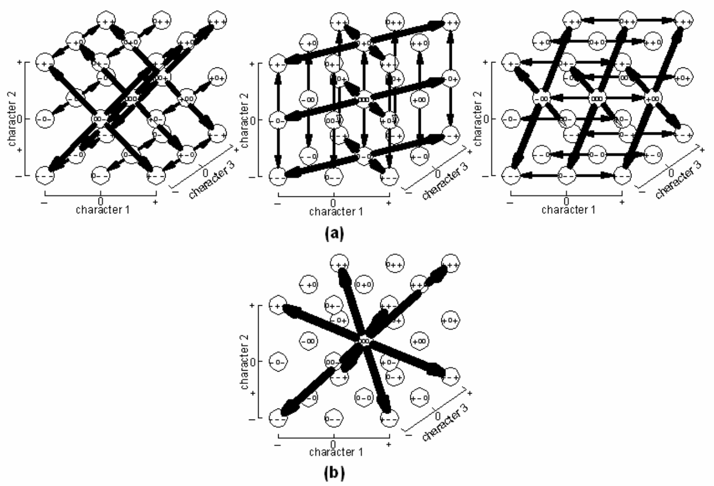


Fig. 7. Construction of the three dominance space

The combination of the three one-dominance spaces, the three two-dominance spaces and the one three-dominance space lead to an overall dominance space which is not visualised since it becomes too complex to display in a two dimensional medium.

A conceptual neighbourhood diagram for QTC_B can now be created by deleting all ‘non-existing’ transitions between relations (edges in the CND) and by deleting all ‘non-existing’ relations (nodes in the CND). The CND for QTC_{B2} is equal to the overall dominance space for three orthogonal qualitative values [16], since all relations and all transitions between relations exist. The CND of QTC_{B1} (Figure 8) is different, since we know that only 17 relations are possible, so we delete the ten impossible relations from the overall dominance space as well as all transitions between impossible and (im)possible relations [20].

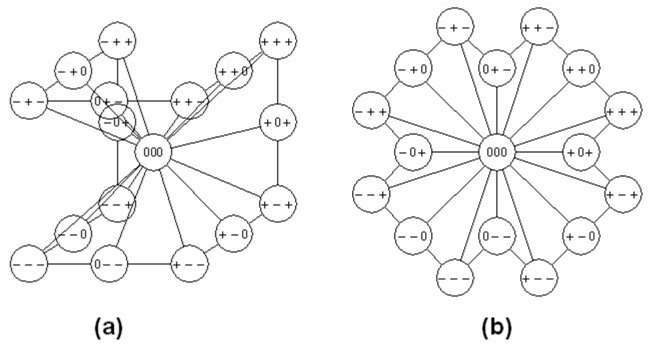


Fig. 8. CND for QTC_{B1}

3 The Qualitative Trajectory Calculus on Networks (QTC_N)

According to Moreira et al. [15] there are two types of moving objects: objects that have a completely free trajectory which is only constrained by the dynamics of the object itself (e.g. a bird flying through the sky), and objects that have a constrained trajectory (e.g. a train on a railway track). Note that QTC_{B2} is able to describe the movement of objects which have a free trajectory in two dimensions, and that QTC_{B1} can describe objects which have a constrained (linear) trajectory. The point is that QTC is not able to handle objects that are moving on a set of interconnected linear features such as a network. Consequently, we develop the qualitative trajectory calculus on networks: QTC_N.

3.1 Network: Some Definitions

A network, such as a road, rail or river network, is a set of interconnected linear features, which can easily be represented by a connected graph. The graph itself is not a spatial structure, but needs to be embedded in a space or must be 'spatialised' [11]. This can be done by a function which maps each node of the graph onto a location in the defined space, and maps each link of the graph onto a curve segment [11]. In essence, a network is a co-dimensional structure. It is a one-dimensional structure embedded in a two-dimensional or three-dimensional space.

We make the following definitions and assumptions:

- a *graph* is a set of edges, *E* and nodes, *N*
- a QTC *network* is a connected graph and a finite set of *objects*
- each edge connects a pair of nodes
- each node has a *degree* which is the number of edges connected to it
- at any time, each object *o* has a *position* in the graph, which is either a node in *N*, or is along an edge *e* in *E*, in which case the network at *t* is augmented with an

additional *dynamic* node of degree 2 cutting the edge e in two, representing the position of o at t.

- a *path* p from o_1 to o_2 at t is a subgraph of the network at time t, such that every node in p is of degree 2 except two nodes representing the position of o_1 and o_2, which are of degree 1. Thus a path is a sequence of nodes and edges from o_1 to o_2.
- every edge has an associated *length,* which is a positive number
- the length of a path is the sum of the length of the edges in the path
- if M is a path of length $|M|$ and there is no path of length less than $|M|$ between the same two nodes, then M is a *shortest path*
- a *cycle* is a subgraph that contains the same number of edges and nodes, and each node is of degree 2[5].

3.2 Formal Definition of QTC$_N$

In QTC$_N$, the landmark used to qualitatively compare the movements of two objects is the shortest path between these objects. This landmark is chosen because of its specificity; a moving object can only approach another object, if and only if it moves along a shortest path between these two objects.

Theorem:

A primary object k *on a network can only decrease its distance to a reference object* l *on this network if and only if* k *moves towards* l *along a shortest path.*

Proof:

1. Moving along a shortest path will decrease the distance

Assume a shortest path between k and l is M, and therefore the shortest distance between k and l is $|M|$. If objects (k or l) move along the shortest path M over an infinitesimal unit of distance (ds), they will decrease their distance because ds is not negative.

$$|M| > |M| - ds \tag{13}$$

2. Moving along any other path (which is not a shortest path) will increase the distance

Assume a shortest path between k and l is $M,$ and therefore the shortest distance between k and l is $|M|$. Any other path N with a length of $|N|$ between k and l, which is not a shortest path, will be longer.

$$|M| < |N| \tag{14}$$

If k moves along N by a distance ds, where $ds < 0.5\ (|N|-|M|)$, then its distance from l will be $|M| + ds$, since N is not a shortest path. So, if k wants to approach l it must move along a shortest path. □

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[5] For technical reasons we require that every cycle in a network have at least 3 non dynamic nodes -- this can always be achieved by adding further nodes of degree 2 without affecting the overall network topology.

Using this property, we can state that an object can only approach another object at time *t* if it does not lie on SP_{kl}^{t} [6] during t^- and lies on SP_{kl}^{t} during t^+[7]. An object moves away from another object if it lies on SP_{kl}^{t} during t^- and does not lie on SP_{kl}^{t} during t^+. If an object lies on SP_{kl}^{t} only at *t* but not during t^- or t^+, or if it lies on SP_{kl}^{t} throughout $[t^-, t^+]$, then the object will be stationary with respect to the other object (although this relation can only last for an instantaneous moment in time).

We can now reformulate conditions (1) to (12) originating for the construction of the three character label for QTC$_B$ to a QTC$_N$ setting.

1. Movement of the first object *k*, with respect to the position of the second object *l* at time point *t*:
   −: *k* moves along the shortest path:
   $$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow k \mid t^- \notin SP_{kl}^{t})) \wedge \\ \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow k \mid t^+ \in SP_{kl}^{t})) \qquad \textbf{(15)}$$
   +: *k* does not move along the shortest path:
   $$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow k \mid t^- \in SP_{kl}^{t})) \wedge \\ \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow k \mid t^+ \notin SP_{kl}^{t})) \qquad \textbf{(16)}$$
   0: stationary:
   $$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow k \mid t^- \in SP_{kl}^{t})) \wedge \\ \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow k \mid t^+ \in SP_{kl}^{t})) \qquad \textbf{(17)}$$
   $$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow k \mid t^- \notin SP_{kl}^{t})) \wedge \\ \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow k \mid t^+ \notin SP_{kl}^{t})) \qquad \textbf{(18)}$$
2. Movement of the second object *l*, with respect to the position of the first object *k* at time point *t* can be described as in Case 1 with *k* and *l* interchanged, hence:
   −: *l* moves along the shortest path
   +: *l* does not move along the shortest path
   0: stationary
3. Relative speed of the first object *k* at time point *t*, with respect to the second object *l* at time point *t*:
   −: *k* is slower than *l*:

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[6] SP_{kl}^{t} denotes the shortest path at time *t* between objects *k* and *l*.

[7] A point *p* lies on a line *L* if it is an element of this line ($p \in L$).

$$v_k \mid t < v_l \mid t \tag{19}$$

+: *k* is faster than *l*:

$$v_k \mid t > v_l \mid t \tag{20}$$

0: *k* and *l* are equally fast:

$$v_k \mid t = v_l \mid t \tag{21}$$

3.3 Possible Relations and Conceptual Neighbours

In order to construct a CND for QTC_N, the possible relations between two objects and the transitions between these relations need to be examined.

If a network is connected, there is always a path between two objects along which they can move towards or away from each other. Since QTC_{B1} describes the movement of objects moving on a line, every binary relation and every transition between these relations stated in QTC_{B1} exists in QTC_N. Every relation in QTC_{B1} can be reached by only changing the speed of the objects. Thus, a transition between relations is triggered by a 'Speed Change' event. Still, due to the co-dimensional nature of a network, there can be additional binary relations and transitions between these relations if the speed is constant. These additional relations and transitions are invoked by two events [2, 17]:

1. a 'Node Pass' event: an object passes a node
2. a 'Shortest Path Change' event: the shortest path between the objects changes.

3.3.1 A Single 'Node Pass' Event

Suppose object *m* approaches object *n* (Figure 9a). Using the definition of QTC_N, object *m* will evoke a '–' in the three character label. If *m* reaches a node in the network with a minimum degree of three, it can either continue its way along a shortest path or it can continue its way on an arc that does not belong to a shortest path. The latter implies that there will be a change in the relation between *m* and *n*, because an object can only approach another object if it moves along a shortest path. At the instantaneous moment in time when *m* passes the node, it will not approach or move away from the other object *n* (Figure 9b). According to the definition (18), *m* will evoke a '0' in the three character label. A fraction of time later, *m* will increase its distance with regard to *n*, evoking a '+' in the three character label defining the relation between objects *m* and *n* (Figure 9c). A 'Node Pass' event can results in a conceptual animation[8] where one of the first two characters in the label changes from '–' to '0' to '+'.

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[8] A conceptual animation is a sequence of QTC relations, following the constraints imposed by qualitative reasoning.

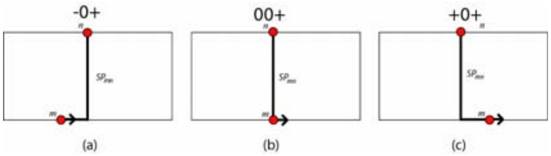

Fig. 9. Example of a transition due to a 'Node Pass' event

In order to have a change in relation due to a 'Node Pass' event certain conditions have to be fulfilled. First of all the degree of the node where the object passes should be at least three. If the degree of the node is less than three and the object moves along the shortest path, an object with positive speed can only continue its way along this shortest path. Secondly, the object causing a 'Node Pass' event must approach the other object. Suppose this object would move away from the other object, it can only continue its way along an arc that does not belong to the shortest path, when it reaches a node. Finally, due to the theory of dominance, a 'Node Pass' event should hold over an interval. Given these three conditions, the transitions caused by a 'Node Pass' event, can be visualised in a CND. Figure 10 gives an overview of all possible transitions between relations due to a single 'Node Pass' event.

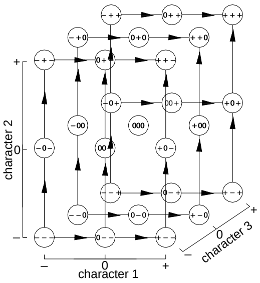

Fig. 10. Possible transitions due to a single 'Node Pass' event

3.3.2 A Single 'Shortest Path Change' Event

We will now illustrate a transition by means of an example shown in Figure 11. Suppose object *m* lies in between nodes B and C. If *m*, B and C lie on a cycle, there are at least two paths between *n* and *m*. One reaches *m* via node B another reaches *m* via node C. In Figure 11a, there is a shorter path via node B (*n*,A,B,*m*) and a longer path via node C (*n*,A,C,*m*). When *m* moves away from this shorter path, and therefore moves away from the other object *n*, *m* will, according to the definition, evoke a '+' in the three character label. This means the shorter path will be extended and the longer path is shortened. At some moment in time, these two paths will become equally long (Figure 11b). At that instantaneous moment, *m* will not approach nor move away from *n*. As a result *m* will evoke a '0' in the three character label. A fraction of time later, *m* will move along the newly defined shortest path and therefore decrease its distance compared to the other object, evoking in a '–' in the three character label defining the relation between *m* and *n* (Figure 11c). A 'Shortest Path

Change’ event can result in a conceptual animation where one of the first two characters in the label changes from ‘+’ to ‘0’ to ‘–‘.

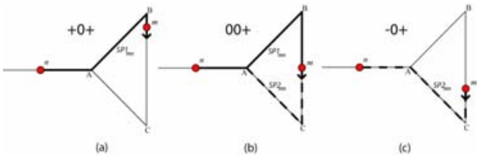

Fig. 11. Example of a transition due to a ‘Shortest Path Change’ event

Here too three conditions should be satisfied in order to have a change in relation due to a ‘Shortest Path Change’ event. First of all, at least one of the objects needs to move away from the other object. If both objects are approaching each other, there cannot be a change of the shortest path and thus there cannot be a change in the relation between both objects if the speed remains positive. Secondly, object *m* needs to lie on a cycle in the network; otherwise *m* can only be reached by paths using the same immediately proceeding node. In order to have a transition in the relations, the immediately proceeding node of *m* changes (e.g. the immediately proceeding node of *m* changes from node B to node C in Figure 11). Finally, due to the theory of dominance, a ‘Shortest Path Change’ event should hold over an interval. Figure 12 gives an overview of all possible transitions between relations due to a single ‘Shortest Path Change’ event.

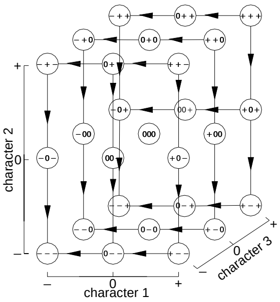

Fig. 12. Possible transitions due to a single ‘Shortest Path Change’ event

3.3.3 Combination of Events

A transition between the QTC$_N$ relations can be caused by three events. Since these three events (‘Speed Change’ event, ‘Node Pass’ event and ‘Shortest Path Change’ event) occur independently and a ‘Node Pass’ event or a ‘Shortest Path Change’ event is caused by only one object, two or more events can occur simultaneously. The result of a transition is the combined transition of the occurring events.

3.3.3.1 A Combined ‘Node Pass’ Event

When object *m* and object *n* both approach each other, it can be that both objects simultaneously pass a node and therefore create the possibility of a combined ‘Node Pass’ event (Figure 13). This property induces three additional conceptual animations, resulting in six new transitions as shown in Figure 14.

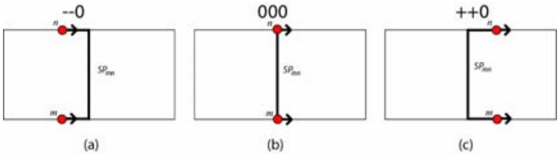

Fig. 13. Example of a transition due to a combined ‘Node Pass’ event

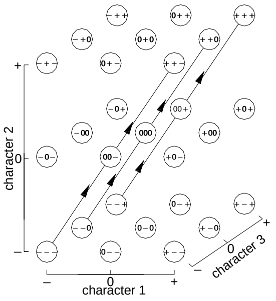

Fig. 14. Possible transitions due to a combined ‘Node Pass’ event

3.3.3.2 A Combined ‘Shortest Path Change’ Event

Assume that object *m* and object *n* both lie on a cycle within the network. Both objects can lie on two different cycles (Figure 15) or the same cycle (Figure 16). When both objects are moving away from each other, there is a possibility that the node closest to each object along the shortest path in the direction of the other object changes when the shortest path changes. This leads to a combined ‘Shortest Path Change’ event. This allows three additional conceptual animations resulting in six new transitions (Figure 18a).

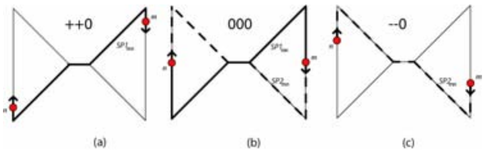

Fig. 15. Example of a transition due to a combined ‘Shortest Path Change’ event when both objects lie on a different cycle

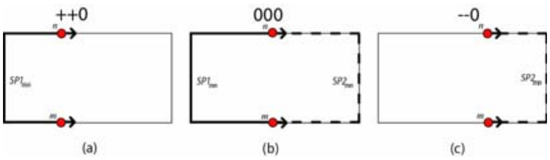

Fig. 16. Example of a transition due to a combined ‘Shortest Path Change’ event when both objects lie on the same cycle

A combined ‘Shortest Path Change’ event can also occur when only one object is moving away from the other object. This transition is exemplified in Figure 17. In Figure 17a both objects lie on the same cycle. This means that there are two paths between object *n* and object *m*. There is one shorter path (*m*,A,B,*n*), and one longer path (*m*,D,C,*n*). When *n* is moving away from *m*, *m* is moving towards *n* and *n* is moving faster than *m*, the shorter path will be extended and the longer path will get shorter. At some moment in time, these two paths will become equally long (Figure 17b). At that instantaneous moment, neighter object will approach or move away from each other. As a result both objects will evoke a ‘0’ in the three character label. A fraction of time later, both objects will move along the newly defined shortest path and therefore *n* will decrease its distance compared to *m* and *m* will increase its distance compared to *n* (Figure 17c). This allows two additional conceptual animations resulting in four new transitions (Figure 18b).

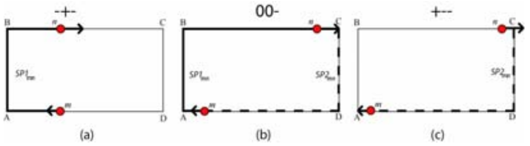

Fig. 17. Example of a transitions due to a combined ‘Shortest Path Change’ event

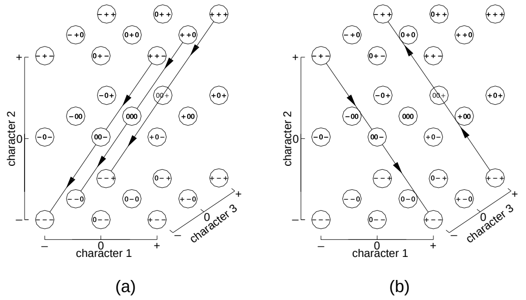

Fig. 18. Possible transitions due to a combined ‘Shortest Path Change’ event

3.3.3.3 A Combination of a ‘Node Pass’ Event and a ‘Shortest Path Change’ Event

Figure 19 illustrates a transition caused by a combination of a ‘Node Pass’ event and a ‘Shortest Path Change’ event. This transition occurs when one object passes a node and simultaneously the shortest path changes due to the other. This transition can only occur if the object that passes a node approaches the other object. The other object must then move away from this object and lie on a cycle of the network. A combination of a ‘Node Pass’ event and a ‘Shortest Path Change’ event allows six additional conceptual animations resulting in twelve new transitions as shown in Figure 20.

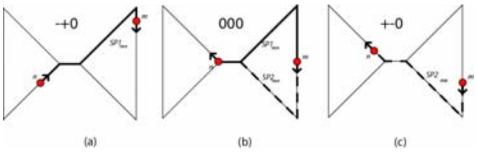

Fig. 19. Example of a transition due to a combined ‘Node Pass’ and ‘Shortest Path Change’ event

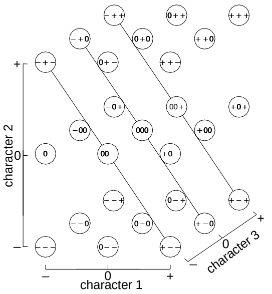

Fig. 20. Possible transitions due to a combined ‘Node Pass’ and ‘Shortest Path Change’ event

3.3.3.4 A Combination of a ‘Speed Change’ Event and a (Combined) ’Node Pass’ Event and/or a (Combined) ‘Shortest Path Change’ Event

Apart from the fact that objects need to move in order for a ‘Node Pass’ event or a ‘Shortest Path Change’ event to occur, speed is independent of these two events. Therefore a ‘Speed Change’ event is also independent of these two events. This means that a ‘Speed Change’ event can occur simultaneously with a single or a combination of ‘Node Pass’ events and/or a single or a combination of ‘Shortest Path Change’ events. An example of a combination of such events is shown in Figure 21. The transitions caused by a combination of a ‘Speed Change’ event and a ‘Node Pass’ event and/or a ‘Shortest Path Change’ event are visualised in Figure 22.

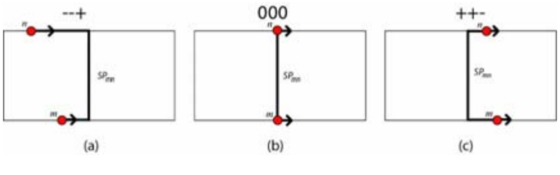

Fig. 21. Example of a transition due to a combination of a ‘Speed Change’ event and a (combined) ‘Node Pass’ event and/or a (combined) ‘Shortest Path Change’ event

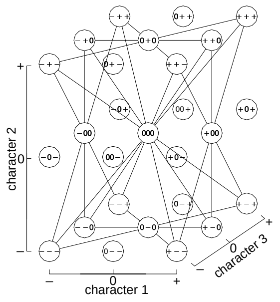

Fig. 22. Possible Transitions due to a combination of a 'Speed Change' event and a (combined) 'Node Pass' event and/or a (combined) 'Shortest Path Change' event

3.4 The Conceptual Neighbourhood Diagram for QTC$_N$

By combining all possible transitions between relations described in 3.3, an overall CND for QTC$_N$ can be constructed. The overall CND is presented in Figure 23. The CND clearly shows that all of the 27 (3^3) theoretically possible relations exist, but not all of them last over an interval. The ten dashed nodes represent relations that can only exist for an instantaneous moment in time. These relations are the ten none-existing relations in QTC$_{B1}$. The CND also reveals that in contrast to the CND for QTC$_{B2}$, not all theoretically possible transitions between relations for QTC$_N$ exist. Out of a possible 98 transitions, 76 remain feasible.

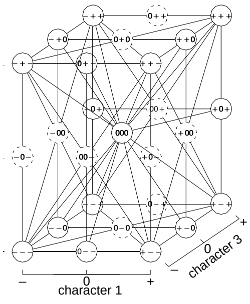

Fig. 23. The Overall CND for QTC$_N$

4 Conclusions and Further Work

In this paper we defined a Qualitative Trajectory Calculus on Networks (QTC_N). A Conceptual Neighbourhood Diagram (CND) was constructed for QTC_N. Note that QTC_N is more expressive than $QTC_{B1,}$ since ten extra relations exist. However, in QTC_N fewer transitions between relations exist than in QTC_{B2}. In a way QTC_N can be positioned somewhere in between QTC_{B1} and QTC_{B2}. This can be explained by the co-dimensional structure of a network. It is a one-dimensional structure embedded in a two-dimensional or three-dimensional space.

We strongly believe that QTC_N is useful in representing moving objects in the framework of a predefined network. Given that nearly all traffic movements are bounded by a network, QTC_N's application field in Geographic Information Systems for Transportation (GIS-T) seems to offer great potential. We plan to evaluate QTC_N in this domain.

We have presented QTC_N in a relatively informal way concentrating on presenting ideas illustrated with simple examples. In Future work we will fully formalize QTC_N. Moreover, we will construct a composition table [1, 3] for QTC_N

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