Representing Moving Objects in Computer-Based Expert Systems: The Overtake Event Example

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Abstract

Qualitative formalisms suited to express qualitative temporal or spatial relationships between entities, have gained wide acceptance as a useful way of abstracting from the real world. The question remains how to describe spatio-temporal concepts, such as the interaction between moving objects, adequately within a qualitative calculus and more specifically how to use this in expert systems. With this in mind, the Qualitative Trajectory Calculus (QTC) has been introduced. QTC enables comparisons between positions of objects at different time points to be made. By reducing the continuum to the qualitative values −, 0 and +, continuous movements can be described qualitatively. To illustrate the naturalness of QTC, the overtake event is studied. An overtake event is a typical example of objects moving in a particular domain and can become important, for example in the study of traffic engineering. A so-called conceptual animation is represented, being a sequence of QTC-relations, following the constraints imposed by qualitative reasoning. It is shown that different kinds of behaviour having certain common characteristics are reflected by the structure (e.g. symmetrical aspects) of the conceptual animations.

Keywords: Moving objects, Qualitative reasoning, Qualitative representation, Spatio-temporal modelling

1. Introduction

Collecting, storing, transforming, analysing and displaying spatio-temporal data is becoming more and more important (Longley, Goodchild, Maguire, & Rhind, 2001). There are many research areas working with spatio-temporal data, such as geography, geology, archaeology, and different socio-economic disciplines, e.g. criminology and transportation. Applications emerging from new technologies include location-based services (mobile phones, GPS), produce huge collections of space-time trajectories. To represent and reason about the complexity of real-world phenomena involving spatio-temporal objects, the representation of relationships in space, time and space-time by means of a computer-based expert system is a mandatory requirement.

In the last two decades, qualitative formalisms suited to express qualitative temporal or spatial relationships between entities have gained wide acceptance as a useful way of abstracting from the real world. Temporal calculi have been proposed, such as the Interval Calculus (Allen, 1983) and the Semi-Interval Calculus (Freksa, 1992a). In the domain of spatial reasoning, Randell, Cui and Cohn (1992) proposed the Region Connection Calculus focusing on topological relationships between regions (see Figure 1). Simultaneously, within the research field of spatial databases, Egenhofer and Franzosa (1991) defined the same set of relations in their 9-Intersection model. Only in recent years has attention been extended to applications that involve spatio-temporal data. Nevertheless, a variety of research communities have been studying movements of objects. Until now, in the database approaches and in expert systems, little attention has been paid to the so-called qualitative aspects (Wolfson, Xu, Chamberlain, & Jiang, 1998; Erwig, Güting, Schneider, & Vazirgiannis, 1999; Moreira, Ribeiro, & Saglio, 1999; Nabil, Ngu, & Shepherd, 2001; Pfoser, 2002). In the domain of robotics (e.g. Stolzenburg, Obst, & Murray, 2002), most systems rely on algorithms, which are from a logical point of view rather ad hoc (Bennett, 1997). Modelling of moving objects has also become a topic of increasing interest in the area of video databases. Fernyhough, Cohn and Hogg (2000) presented a technique to learn traffic event types from a visual input expressed in a qualitative calculus. In principle the complete overtake event – which will be analysed in detail in this paper – could have been learnt, but was not in fact since the observed area was not large enough. The question remains how to describe motion, and more specifically the interaction between moving objects, adequately within a qualitative calculus. Van de Weghe (2004) presents the Qualitative Trajectory Calculus (QTC), which is a theory for representing and reasoning about movements of objects in a qualitative framework, differentiating groups of disconnected objects. To our knowledge, QTC has never been used in an expert system environment.

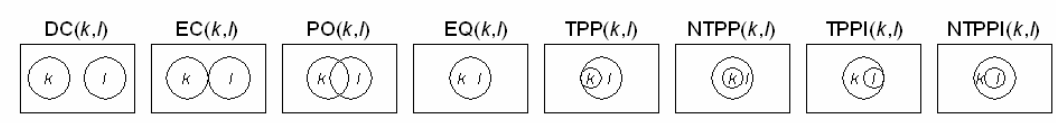

Fig. 1: Relations in the Region Connection Calculus
(source: Randell, Cui, & Cohn, 1992).

This paper is structured as follows. First, we define the problem being studied: i.e. how to present and model moving objects. Next, we introduce QTC as potential tool to make such movements explicit in an expert system. Third, we illustrate the usefulness of QTC by means of an example. We conclude with our major findings and point to some avenues for future research.

2. Problem setting: representing and modelling moving objects

2.1. Moving objects

Change is a central concept in the spatio-temporal domain (Muller, 1998; Galton, 2000). An important classification of changes is based on the difference between discontinuous and continuous changes. In discontinuous changes, in any sufficiently small temporal neighbourhood, an attribute having ordered values will change from one value to another without taking on all intermediate values. On the other hand, continuous change occurs when all intermediate attribute values are passed through.

Stated in the words of Galton (1995, p.377) "the phenomenon of movement arises whenever the same object occupies different positions in space at different times". Everyone has an intuitive idea of continuous motion, such as the path taken by a thrown ball (Mortensen, 1999). A motion is often represented by a trajectory, which is a connected non-branching continuous line having a certain shape and direction (Eschenbach, Habel, & Kulik, 1999).

2.2. Qualitativeness

Reasoning can be performed on quantitative as well as qualitative information. Typically when working with quantitative information, a predefined unit of a quantity is used (Goyal, 2000). For example, one could say that the distance between two cities is 55 kilometres; hence making the kilometre the predefined unit. In the qualitative approach, continuous information is quantised or qualitatively discretised by landmarks separating neighbouring open intervals, resulting in discrete quantity spaces (Weld & de Kleer, 1990). The major idea in the qualitative approach is that a distinction is introduced only if it is relevant to the problem (Cohn, 1996; Clementini, Di Felice, & Hernandez, 1997). Thus, qualitative reasoning only studies the essence of information, represented as a small set of symbols such as the quantity space {–, 0, +} consisting of the landmark value "0" and its neighbouring open intervals "–" and "+". For example, if one does not know the precise speed of a car and a bicycle, but knows that the speed of the car is higher than the speed of the bicycle, one can label this with the qualitative value +, meaning that the car is moving faster than the bicycle. One could also say that the bicycle is moving slower than the car, by giving the qualitative value – to this relation. Finally, both objects can also move at the same speed, resulting in a qualitative value 0. One thing is for sure; the speed of a car cannot change from being higher than the speed of the bicycle to being lower than the speed of the bicycle, without passing the value 0. This seemingly rather straightforward idea of continuity is of vital importance to qualitative reasoning (Cohn & Hazarika, 2001) and is clearly described by Forbus (1990, p.25): "*Continuity is a formal way of enforcing the intuition that things change smoothly. A simple consequence of continuity, respected by all systems of qualitative physics, is that, in changing, a quantity must pass through all intermediate values. That is, if* $A < B$ *at time* t_1*, then it cannot be the case that at some later time* t_2 $A > B$ *holds, unless there was some time* t_3 *between* t_1 *and* t_2 *such that* $A = B$".

Qualitative information may be more available and less expensive to obtain than quantitative information (Freksa, 1992a). By abstracting away from metrical details, qualitative representations are also much more appropriate for handling incomplete information (Cristani, Cohn, & Bennett 2000). In addition, it has been recognised that a qualitative approach is more appropriate for the representation of spatio-temporal human cognition than a quantitative calculus (Sharma, 1996). Therefore, we introduced the Qualitative Trajectory Calculus (QTC), which is a qualitative calculus for representing moving objects.

3. Qualitative Trajectory Calculus (QTC)

Depending on the level of detail and the number of spatial dimensions, different types of QTC are defined in Van de Weghe (2004), all belonging to QTC - Basic (QTC_B) or QTC - Double-Cross (QTC_C). In this paper, we focus on two-dimensional movements. In general, QTC makes comparisons between positions of objects at different time points. The movement of the first object (k) with respect to the second object (l) is studied by comparing the distance between l at the current time point (t) and k at the time points immediately before the current time point (t^-), with the distance between l at t and k at the time points immediately after the current time point (t^+). On the other hand, the movement of l with respect to k is studied by comparing the distance between k at t and l at t^-, with the distance between k at t and l at t^+. Each object can move away from or towards the other, or can be stable with respect to the other, respectively resulting in a qualitative value of +, – or 0. In QTC_B, only this changing distance between two objects is of importance. Each trajectory pair is thus represented in QTC_B by one two-tuple out of a set of nine ($=3^2$) possibilities. Figure 2 depicts these so-called QTC_B-relation icons.

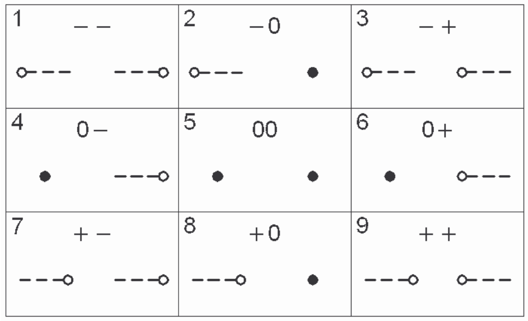

Fig. 2: QTC_B-relation icons.

QTC_C also considers the direction in which an object is moving with respect to the reference line between both objects. As a direct result, QTC_C gives much more detail than QTC_B, with an increased complexity. QTC_C, which is partly based on the Double-Cross Calculus introduced by Freksa and Zimmerman (Freksa, 1992b; Zimmermann & Freksa, 1996), examines the movement of two objects k and l with respect to each other, between t_1 and t_2. Both movements are represented via a vector (Figure 3(b)), degenerated to a point if an object is not moving (Figure 3(a)). The origins of these vectors serve as pinpoints for the double-cross, being the reference frame for the calculus. Through these pinpoints, the reference line (*RL*) is defined. Also through these pinpoints and perpendicular to *RL*, the first perpendicular reference line ($RL\perp1$) and the second perpendicular reference line ($RL\perp2$) are defined.

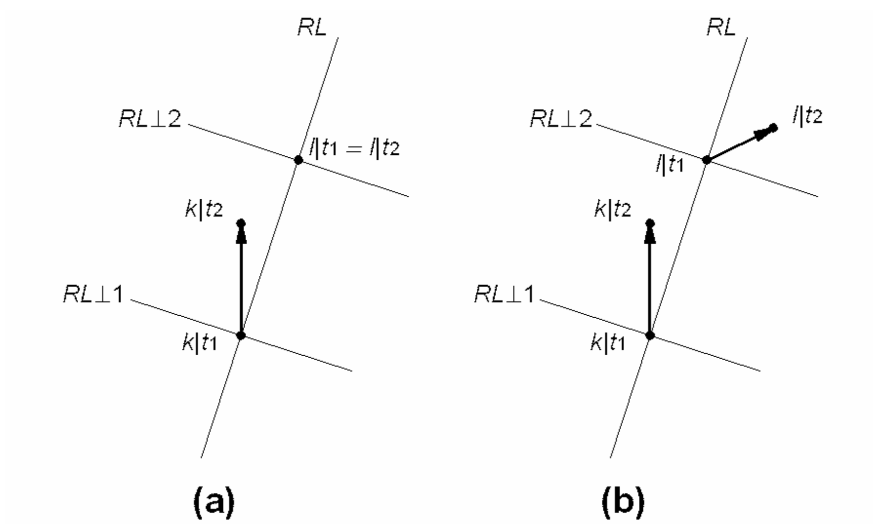

Fig. 3: Reference frame for QTC_C.

These three lines form a so-called double-cross. In QTC_C, two dichotomies form the basis of the reference frame: the towards/away-from dichotomy and the left/right dichotomy. The first dichotomy results in the first and the second character of the

QTC_C label. The second dichotomy also results in two characters: the third and the fourth character of the QTC_C label. By reducing the continuum to the qualitative values –, 0 and +, the underlying continuous system can be described quite qualitatively. Hence, a two-dimensional movement is presented in QTC_C using the following functions:

1. Movement of the first object with respect to the first perpendicular reference line at time point *t*:

   –: *k* is moving towards *l*

   +: *k* is moving away from *l*

   0: *k* is stable with respect to *l*

2. Movement of the second object with respect to the second perpendicular reference line at time point *t*:

   –: *l* is moving towards *k*

   +: *l* is moving away from *k*

   0: *l* is stable with respect to *k*

3. Movement of the first object with respect to the directed reference line from *k* to *l* at time point *t*:

   –: *k* is moving to the left side of *RL*

   +: *k* is moving to the right side of *RL*

   0: *k* is moving along *RL*

4. Movement of the second object with respect to the directed reference line from *k* to *l* at time point *t*:

   –: *l* is moving to the left side of *RL*

   +: *l* is moving to the right side of *RL*

   0: *l* is moving along *RL*

We can represent a trajectory by a label consisting of four characters giving a value for the four functions above. As presented in Figure 4, there are 81 ($=3^4$) QTC_C-relations. The left dot represents the position of k and the right dot the position of *l*. A dot is filled if the object can be stationary, and open if the object cannot be stationary. The quarter parts of circles stand for an open polygon, for which each line segment drawn from the object point to the curved side of the quarter part stands for a possibility. It is important that the polygons are open, i.e. the movement of k in relation $(- - - -)_C$ in Figure 4 can be from k to every point on the curved part of the quarter part excluding the horizontal and the vertical line segments.

Fig. 4: QTC_C-relation icons.

Having introduced the basic notation for QTC, we now turn our attention to the practical relevance of this approach for computer-based expert systems. To this end, the overtake event is used.

4. The overtake event

Ever since Vickrey's (1969) idea on congestion modelling and traffic queues, an increasing number of traffic engineers have tried to represent and understand the complex physics associated with moving (car) objects. Obviously, if expert systems are to be a help in understanding these complex traffic relations, a proper semiology has to be introduced. Here, we think that QTC_C is useful.

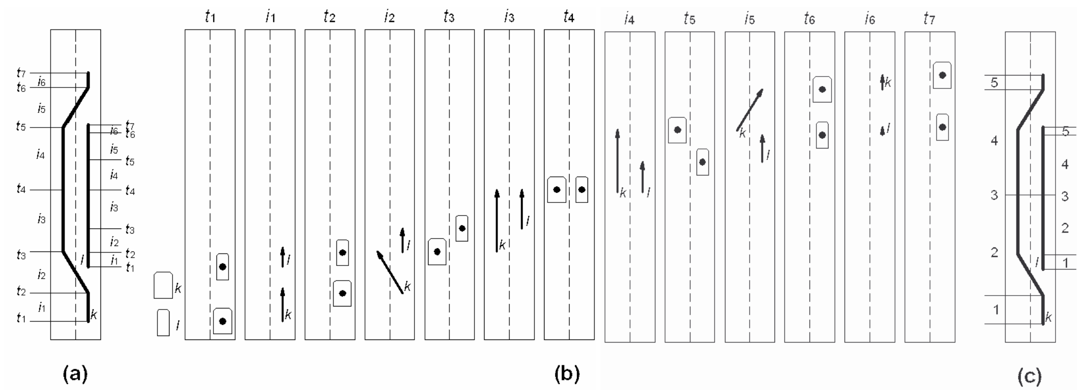

Fig. 5: Representation of an overtake event in QTC_C.

It is obvious that more than one snapshot is needed to be able to study the whole overtake event. Some QTC_C-relations may hold over an instantaneous time point (t_i) while others can only hold over an interval (i_i). Figure 5(a) represents the movement of two cars (*k* and *l*) during an overtake event, composed out of seven time points. Figure 5(b) goes more in detail:

- movements at t_1, t_2, t_3, t_4, t_5, t_6, and t_7, representing the positions of both cars at the specific time point;
- movements during intervals i_1, i_2, i_3, i_4, i_5, and i_6, representing the travelled path during the specific interval.

The following changes in the trajectories of one of both cars or in the interaction between both cars are expressed in QTC_C:

- Ex|i_1: car *k* and car *l* are both driving in the first traffic lane, *k* is driving behind *l*.
- Ex|i_2: car *k* is heading out to the second lane.
- Ex|i_3: car *k* is driving in the second lane and is driving behind car *l*, which is driving in the first lane.

- Ex|t_4: car k is driving in the second lane and is passing car l, which is driving in the first lane.
- Ex|i_4: car k is driving in the second lane and is driving in front of car l, which is driving in the first lane.
- Ex|i_5: car k is heading back to the first lane.
- Ex|i_6: car k and car l are both driving in the first lane, l is driving behind k.

Note that, apart from "Ex|t_4: car k is driving in the second lane and is passing car l, which is driving in the first lane", all expressions hold during an interval having certain duration. The QTC$_C$-relations during these intervals can be read from Figure 5(b) without any difficulty:

$$i_1(-\ +\ 0\ 0)_C,\ i_2(-\ +\ -\ +)_C,\ i_3(-\ +\ -\ +)_C,\ i_4(+\ -\ -\ +)_C,\ i_5(+\ -\ -\ +)_C,\ i_6(+\ -\ 0\ 0)_C$$

Clearly, an overtake event is only a part of a longer period of two cars driving on a road. Therefore, we may say that the relations during i_1 and i_6 can be extrapolated to respectively immediately before t_1 and immediately after t_7, and thus $t_1(-\ +\ 0\ 0)_C$ and $t_7(+\ -\ 0\ 0)_C$ holds.

An interesting thing happens at t_2. Immediately before t_2 (at t_2^-) k is moving along RL, and immediately after t_2 (at t_2^+) k is moving to the left side of RL. According to Forbus' equality change law (1984) and Galton's theory of dominance (1995; 2000) an interval-like qualitative value (+ or − in QTC$_C$) will reach a landmark value (0 in QTC$_C$) at a time instant. Since $i_1(-\ +\ 0\ 0)_C$ and $i_2(-\ +\ -\ +)_C$, the third character of the description at t_2 must be 0 rather than −, and the fourth character of the description at t_2 must be 0 rather than +, resulting in $t_2(-\ +\ 0\ 0)_C$. Hence, dual reasoning applies for:

- t_6 is between $i_5(+\ -\ -\ +)_C$ and $i_6(+\ -\ 0\ 0)_C$.

  Thus, the third character is 0 and the fourth character is 0.

  So, $t_6(+\ -\ 0\ 0)_C$.

A transition where a character changes directly from + to − (or vice versa) without taking on the intermediate value of 0 would be discontinuous. Therefore, the solution is to note that there is an intermediate value at an intermediate time point, resulting in the qualitative value 0 at this intermediate time point. This can be applied to:

- t_4 is between $i_3(-\ +\ -\ +)_C$ and $i_4(+\ -\ -\ +)_C$.

  Thus, the first character is 0 and the second character is 0.

  So, $t_4(0\ 0\ -\ +)_C$.

After this, the remaining time point relations can de determined without difficulties, governed by the descriptions of the intervals around them:

$$t_3(-\ +\ -\ +)_C \text{ and } t_5(+\ -\ -\ +)_C$$

Finally, a so-called conceptual animation can be represented, being a sequence of QTC_C-relations, following the constraints imposed by qualitative reasoning:

$$\{t_1(-\ +\ 0\ 0) \rightsquigarrow i_1(-\ +\ 0\ 0) \rightsquigarrow t_2(-\ +\ 0\ 0) \rightsquigarrow i_2(-\ +\ -\ +) \rightsquigarrow t_3(-\ +\ -\ +) \rightsquigarrow i_3(-\ +\ -\ +) \rightsquigarrow t_4(0\ 0\ -\ +) \rightsquigarrow i_4(+\ -\ -\ +) \rightsquigarrow t_5(+\ -\ -\ +) \rightsquigarrow i_5(+\ -\ -\ +) \rightsquigarrow t_6(+\ -\ 0\ 0) \rightsquigarrow i_6(+\ -\ 0\ 0) \rightsquigarrow t_7(+\ -\ 0\ 0)\}_C$$

Thus, the overtake event can be characterised by the following sequence of QTC_C-relations (note that only the third temporal primitive $((0\ 0\ -\ +)_C)$ is instantaneous):

$$\{(-\ +\ 0\ 0) \rightsquigarrow (-\ +\ -\ +) \rightsquigarrow (0\ 0\ -\ +) \rightsquigarrow (+\ -\ -\ +) \rightsquigarrow (+\ -\ 0\ 0)\}_C$$

Let us briefly characterise every QTC_C-relation in the event by a number representing its position: 1$(-\ +\ 0\ 0)_C$, 2$(-\ +\ -\ +)_C$, 3$(0\ 0\ -\ +)_C$, 4$(+\ -\ -\ +)_C$, and 5$(+\ -\ 0\ 0)_C$. This way, one can represent the event as Figure 5(c).

What would happen if we consider right-hand driving in Continental Europe (CE), versus left-hand driving in the United Kingdom (UK)? The UK overtake event is clearly 'symmetrical' with respect to the CE overtake event. Only the last two characters are inverted, the first two remain. Therefore, 1 and 5 remain the same:

- CE overtake event:

$$\{(-\ +\ 0\ 0) \rightsquigarrow (-\ +\ -\ +) \rightsquigarrow (0\ 0\ -\ +) \rightsquigarrow (+\ -\ -\ +) \rightsquigarrow (+\ -\ 0\ 0)\}_C$$

- UK overtake event:

$$\{(-\ +\ 0\ 0) \rightsquigarrow (-\ +\ +\ -) \rightsquigarrow (0\ 0\ +\ -) \rightsquigarrow (+\ -\ +\ -) \rightsquigarrow (+\ -\ 0\ 0)\}_C$$

Dual reasoning applies for a situation where two cars drive on the same road, on different lanes and in different directions. They drive towards each other, pass each other and then drive away from each other. Again, only the two last characters need to be inverted:

- CE passing event:

$$\{(-\ -\ +\ +) \rightsquigarrow (0\ 0\ +\ +) \rightsquigarrow (+\ +\ +\ +)\}_C$$

- UK passing event:

$$\{(- - - -) \rightsquigarrow (0\ 0 - -) \rightsquigarrow (+ + - -)\}_C$$

It is also possible to detect other events representing different kinds of behaviour, e.g. a situation of two cars (*k* and *l*) at a crossroad (see Figure 6).

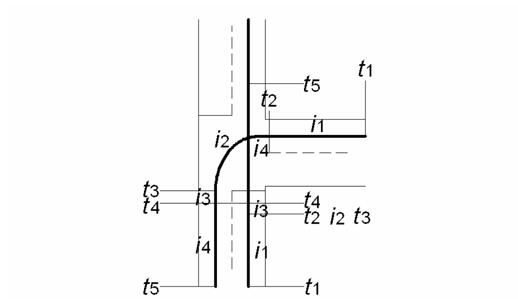

Fig. 6: Representation of the movement of two cars at a crossroad.

Analogous reasoning to the overtake event results in the following conceptual animation:

$$\{t_1(- - + -) \rightsquigarrow i_1(- - + -) \rightsquigarrow t_2(- 0 + 0) \rightsquigarrow i_2(- 0 + 0) \rightsquigarrow t_3(- 0 + 0) \rightsquigarrow$$
$$i_3\ (- - + +) \rightsquigarrow t_4(0\ 0\ 0\ 0) \rightsquigarrow i_4(+ + - -) \rightsquigarrow t_5(+ + - -)\}_C$$

Or, for short:

$$\{(- - + -) \rightsquigarrow (- 0 + 0) \rightsquigarrow (- - + +) \rightsquigarrow (0\ 0\ 0\ 0) \rightsquigarrow (+ + - -)\}_C$$

These basic events in turn can form the basis of compound events. This way, a hierarchical network of events can be generated. We call this concept hierarchical event modelling. We believe it may be useful for event recognition and for qualitative simulation in expert systems. For example, if only five events are allowed in a specific application area, then the simulation may only follow these events.

5. Conclusion and discussions

QTC allows the representation of the interaction between moving objects within a qualitative calculus, and we have indicated how this might be used in expert systems.

Although there exist several approaches to visual query languages for spatial databases, most of them allow querying only static spatial situations. There are only a few approaches to querying image sequences. However, the goal of these proposals is mainly to facilitate queries on video databases and not the querying of spatio-temporal databases. For example, Erwig and Schneider (2003) present a visual notation that is able to describe scenarios of changes in the topological relationships between different objects. A possible avenue for future work would be to extend these topological relationships with the expressiveness afforded by QTC relations. For example, consider extending *spatial-query-by-sketch* (Egenhofer, 1997) so that a user could draw trajectories that are automatically translated to a QTC representation. Such a tool could certainly exploit the expressiveness of QTC.

In the future, the simple overtake example will be applied in practice to some real data. This way, the question of the complexity of data acquisition also needs to be addressed. Moreover, the potential for conversion between data stored using current (quantitative) techniques and QTC have to be studied. Since the amount of data that needs to be handled will increase exponentially, data-reduction is important. This may be achieved by spatio-temporal pre-processing techniques used to achieve data-reduction without information loss, based on the work of spatial (Rodríguez, Egenhofer, & Blaser, 2003) and temporal (Rodríguez, Van de Weghe, & De Maeyer, 2004) data-reduction.

6. Acknowledgements

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