Physical Objects, Identity and Vagueness

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Abstract

The paper presents an approach to constructing an ontology of physical objects founded on a theory of the spatio-temporal distribution of matter types. The starting point for this construction is the formal theory $\mathfrak{O}$ of 'space, time, matter and things' given in (Bennett 2001c), in which physical objects are modelled as maximal self-connected portions of some matter type. However, that theory is not compatible with the commonsense view of 'physical objects' according to which it is normal to regard an object as persisting (i.e. retaining its identity) despite the loss (or possibly gain) of small parts, which are deemed 'insignificant'. The current paper gives a more elaborate theory of the nature of physical objects and specifies their identity criteria in a way that allows for possible loss of small particles. This is achieved by explicitly taking account of an intrinsic vagueness in the identity criteria for commonsense physical objects. The paper also uses this theory to characterise various degrees of physical damage which an object can undergo.

1 Introduction

Research into representing and reasoning with spatial information has produced a number of logical languages that allow spatio-temporal situations and events to be represented at various levels of abstraction and using a variety of different conceptual vocabularies (Randell, Cui and Cohn 1992, Asher and Vieu 1995, Borgo, Guarino and Masolo 1996, Borgo, Guarino and Masolo 1997, Pratt and Schoop 1998, Bennett, Cohn, Torrini and Hazarika 2000b). The relationship of the abstract spatial regions described by these theories to the physical objects of everyday experience is a subject that has received relatively little attention. Some formalisations do exist (e.g. (Davis 1990, Davis 1993, Shanahan 1995)) but they are surprisingly sparse in the KR literature.

A semantics which builds a theory of matter and its evolution through time onto a theory of spatial regions was given in (Bennett 2001c). However the analysis of that paper makes the assumption that physical bodies can be modelled as maximal well-connected regions of some particular matter type. But if we consider the behaviour of real world objects we soon realise that this assumption is an idealisation which cannot give a comprehensive account.

We often describe the world in terms of regular geometrical shapes. For instance, a wooden block may be described as being cubic in shape, even though when we look at it closely we find that its surface is ridged and uneven. It would be useful to have an theory which both gives an accurate ontology of the properties of real physical objects and also accounts for the value of idealising approximations in descriptions of everyday situations.

A related problem concerns the identification of an object at different times, where the object has suffered some form of damage or other alteration during the intervening period. For instance we might want to say that the same statue that was once in an ancient temple is now in a museum, even though its arms and legs are now missing. This problem becomes especially acute when we want to deal with the issue of what happens when physical objects are subject to the loss of tiny insignificant parts. Intuitively, physical objects persist despite that loss; but in order to formalise this we need to have an account of the vague notion of an 'insignificant part'. The the need for a theory of physical objects to take account of fundamental issues of vagueness has been recognised by a number of philosophical studies such as (Hughes 1986) and (Heller 1991). In the present paper I address these issues within a fully formal framework.

In reasoning about physical systems such a mechanical devices we often want to represent situations in which some component is broken or worn away so as to cause a malfunction. The paper concludes with a sketch of how the proposed theory of identity criteria for physical objects can be used to describe damaging events of this kind.

2 The Theory $\mathfrak{O}$ and its Idealised Objects

The analysis of this paper is conducted within the formal theory $\mathfrak{O}$, as presented in (Bennett 2001c). This is a logic specifically designed for ontology construction. I shall now briefly recap the basic syntax and semantics of $\mathfrak{O}$.

2.1 Syntax and informal semantics for $\mathfrak{O}$

$\mathfrak{O}$ contains constant symbols of five types:

- Spatial regions: $\emptyset, \ldots, r_i, \ldots$
- Time points: $\ldots, t_i, \ldots$
- Mass nouns: $\ldots, \mathbf{m}_i, \ldots$
- Individuals: $a, b, c, \ldots$
- Count nouns: $\ldots, \mathrm{C}_i, \ldots$

The region constants denote regular open subsets of the universe $\mathcal{U}$ of spatial points (which is the classical Cartesian space — $\mathcal{U} = \Re^3$) and $\emptyset$ denotes the 'empty region' (i.e. the null set of points).

Time is modelled as a set of 'histories' each of which is a mapping from time points to *states* of the universe. The semantic interpretation function of $\mathfrak{O}$ evaluates propositions relative to some history and time point (which are regarded as the 'actual history' and 'actual time'). A history/time pair $\langle h, t \rangle$ is called an *index*.

Each state is associated with a mapping from the set of mass nouns to regular open subsets of the universe. This mapping gives the spatial extension in that state of the *matter type* referred to by each mass noun.

Each individual constant is associated with an individual function which is a mapping from each index to a regular open subset of $\mathcal{U}$. This gives the extension of that individual in that history and time point corresponding to the index. An individual may be said to *exist* at all indices for which its associated individual function takes a non-null value.

The treatment of count nouns is essentially that of Gupta (1980). At each index a count noun denotes a set of individual functions, subject to the condition that each of these functions has a non-null value at that index — i.e. they correspond to individuals which exist at that index.

The atomic propositions of $\mathfrak{O}$ take the following forms:

$$\alpha = \beta, \quad \mathsf{P}(\alpha, \beta), \quad \mathsf{S}(\alpha), \quad t_1 = t_2, \quad t_1 \leq t_2, \quad \mathsf{AT}(t_1)$$

Where α and β are region terms which may be: either a region variable r_i, a term of the form $\mathsf{ext}(a)$ or $\mathsf{ext}(\mathbf{m})$ or the empty region constant $\emptyset$. $\mathsf{ext}(a)$ is a logical function whose value is the spatial extension of the individual a at the index of evaluation — this is simply the value at that index of the individual function associated with a. Similarly, $\mathsf{ext}(\mathbf{m})$ gives the extension of the matter type referred to by $\mathbf{m}$ (at the index of evaluation). The theory is axiomatised so that $\mathsf{ext}(\mathbf{m})$ can only vary *continuously* with time.

P is the spatial *part* relation, S is the *sphere* predicate, $\leq$ is a (dense, linear) temporal ordering on the time points and $\mathsf{AT}(t_1)$ is true just in case t_1 denotes the 'actual time' (i.e. the time of the index at which the constant is being evaluated by the semantic interpretation function). The equality symbol asserts identity either between two region or two time terms.

As explained in (Bennett 2001c) the spatial predicates P and S are constrained to satisfy a slightly modified version of the Region-Based Geometry (**RBG**) theory (Bennett, Cohn, Torrini and Hazarika 2000c, Bennett et al. 2000b, Bennett 2001b). The modification simply adds the empty region denoted by $\emptyset$ to the domain of quantification.

As well as the usual Boolean connectives, the language of $\mathfrak{O}$ contains the following constructs. If φ and ψ are propositions then so are the following:

$$\circledast\varphi, \quad \mathsf{Holds\text{-}At}(\varphi, t), \quad \forall r[\varphi], \quad \forall t[\varphi], \quad (\forall \mathrm{C}, a)[\varphi]$$

The formula $\circledast\varphi$ asserts that φ holds for all indexes $\langle h', t \rangle$, where t is the actual time and h' is any history which coincides with the actual history for all times up to t. This means that the truth of φ is determined by the actual and past states of the universe and is independent of whatever possible future might transpire.

$\mathsf{Holds\text{-}At}(\varphi, t)$ asserts that formula φ is true at the time point denoted by t (in the actual history). The $\mathsf{Holds\text{-}At}$ relation allows definition of a wide range of temporal operators:

D1) $\mathbf{P}\varphi \equiv_{def} (\exists t_1, t_2)[\mathsf{AT}(t_1) \wedge t_1 > t_2 \wedge \mathsf{Holds\text{-}At}(\varphi, t_2)]$

D2) $\lhd\varphi \ \equiv_{def}\ \exists t_1, t_2)[\mathsf{AT}(t_2) \wedge (t_1 < t_2) \wedge \forall t_3[(t_1 \leq t_3 < t_2) \rightarrow \mathsf{Holds\text{-}At}(\varphi, t_3)]]$

P is a standard past tense operator, $\lhd\varphi$ means that φ is 'just past' in the sense that it holds for some open time interval immediately preceding (i.e. bounded by) the present time. Defining **F** and $\rhd$, the future versions of these operators just requires reversing the directions of the inequalities between time variables.

In the current paper we shall not be greatly concerned with the structure of possible histories but we will want to say that certain things are necessarily true — i.e. true at all index points. This can be expressed by the defined $\Box$ operator:

D3) $\Box\varphi \ \equiv_{def}\ \forall t[\mathsf{Holds\text{-}At}(\boxast\forall t'[\mathsf{Holds\text{-}At}(\varphi, t')], t)]$

The quantifiers $\forall r[\varphi]$ are $\forall t[\varphi]$ range respectively over regions and time points. The sorting is indicated by the variable names. Quantification over individuals is always mediated by a count noun. In evaluating the construct $(\forall \textsc{c}, a)[\varphi]$ at any index, the variable a ranges over all individual functions in the set denoted by $\textsc{c}$ at that index.

The extension of an individual varies from index to index. Moreover it can happen that at some index two different individuals have the same extension.

D4) $a \doteq b \ \equiv_{def}\ (\mathsf{ext}(a) = \mathsf{ext}(b))$

Identity between individuals requires the that they have the same extension at all indices, so, in general individual constants a and b are inter-substitutable only if they satisfy the stronger relation $\Box(a \doteq b)$.

The semantics of individuals means that each individual is directly associated with its own (trans-world) identity criteria, as given by the corresponding individual function. The identity criteria for different types of individual are articulated by axioms characterising the meanings of count nouns. Following Gupta (1980), I define two ways in which a count noun can be predicated of an individual:

D5) $\textsc{c}(\alpha) \ \equiv_{def}\ (\exists \textsc{c}, x)[x \doteq \alpha]$,
D6) $\textsc{c}[\alpha] \ \equiv_{def}\ (\exists \textsc{c}, x)[\Box(x \doteq \alpha)]$.

The first (weaker) predication says that α has the same extension as something that is a $\textsc{c}$ (for example 'That quantity of metal is a ship.'), whereas the second states that α is necessarily identical to some $\textsc{c}$ and hence satisfies whatever identity criteria are common to $\textsc{c}$s (for example 'Titanic is a ship.').

2.2 Chunks and Bodies

One of the main results of (Bennett 2001c) is the precise definition and axiomatisation of the complex count noun $\textsc{chunk-of}(\mathbf{m})$. The individuals satisfying this predicate correspond to maximal self-connected portions of the matter type $\mathbf{m}$.

The semantics of chunks given in (Bennett 2001c) is based on two key assumptions; namely that:

1. chunks are maximal self-connected regions of some matter type;
2. the extensions of matter types can only change continuously.

These conditions cannot be simultaneously satisfied at a point in time where a chunk breaks into pieces or merges with another chunk. Thus, the definition of chunk is such that: when a chunk breaks into two or more parts it ceases to exist immediately before the break point and new chunks corresponding to pieces of the original chunk only come into being immediately after the break. Similarly, if two or more chunks merge together the original chunks cease to exist at the point of merger and a new chunk comes into being.

This analysis is perfectly fine as long as we realise that these 'chunks' are theoretical entities which do not directly correspond to the the objects referred to in everyday communication. When we talk of a cup or a brick we are referring to an object that continues to exist even when it is chipped or scratched. Hence we need to characterise types of object which persist through the loss of certain pieces. To construct a rigorous ontology of commonsense physical objects one needs a theory which, although it must somehow relate them to the matter from which they are formed, does not simply reduced them to idealised chunks.

To bridge the gap between these very different types of object a number of theoretical sortal types will be introduced. Specifically: *rigid sub-chunks* are parts of rigid chunks which are fixed with respect to the extension of a moving chunk; and *rigid matter portions*[1] are then defined to correspond to pieces of chunks which can persist through breaking and merging events involving the chunks in which they subsist. These concepts facilitate the definition of a generic sort of *mesoscopic rigid physical object*, characterising objects whose identity criteria are robust in the face of loss of *insignificant* pieces. Here the notion of 'insignificant piece' will characterised in terms of a *maximal*

[1] What I am calling 'rigid matter portions' correspond with what Davis (1993) calls 'chunks'; and my rigid chunks correspond roughly with his 'solid objects'.

insignificant sphere, **i**. Natural sortals describing types of physical object (e.g. cups or chairs) can then be defined as sub-types of mesoscopic rigid physical object satisfying appropriate qualitative predicates.

3 Rigid Matter, Breakage and Pieces

In this section I shall give a completely formal specification of the notion of a 'piece' of a chunk of rigid matter. Intuitively this may seem straightforward but in fact getting a completely precise definition is rather complex given that the matter forming a chunk may move in space and may break up or merge with other chunks of matter. We will need to specify identity criteria of pieces which enable us to (for example) trace a fragment of pottery back to an original intact cup via a series of breakages.

3.1 Rigid Matter

In the current work I focus on *rigid* objects.[2] In (Bennett 2001c) the predicate $\mathsf{Rigid}(a)$ was defined to mean that a exists only during some open time interval and all its extensions at any time point within this interval are congruent. I now define $\mathsf{RigidMatter}(\mathbf{m})$ to say that $\mathbf{m}$ is a type of rigid matter — i.e. all chunks of matter type $\mathbf{m}$ are rigid:

$$\mathbf{D7)}\ \ \mathsf{RigidMatter}(\mathbf{m}) \ \equiv_{def} \ (\forall \textsc{chunk-of}(\mathbf{m}), c)[\mathsf{Rigid}(c)]$$

I also introduce a matter type constant **rm** of rigid matter. This constant satisfies the following axioms:

$$\mathbf{A1)}\ \ \mathsf{RigidMatter}(\mathbf{rm})$$
$$\mathbf{A2)}\ \ \mathsf{RigidMatter}(\mathbf{m}) \rightarrow \Box(\mathsf{P}(\mathsf{ext}(\mathbf{m}), \mathsf{ext}(\mathbf{rm})))$$

In the current work I shall assume that rigid matter is also continuous, which given its rigidity, means that it cannot be created or destroyed (though it can be smashed up and joined together).[3]

3.2 Chunk Extensions at Breakage Points

Although chunks themselves cannot exist at breakage and merger points, nevertheless, for any object that exists over an open time interval we can define its limiting extensions at each end of the interval as follows:

[2]Analysing portions of non-rigid matter (e.g. liquids) would require a rather different system of ontological concepts (see e.g. (Hayes 1985)).

[3]A more comprehensive theory would have to take account of phase changes (freezing, melting, setting etc.) during which rigidity could be acquired or lost.

$$\mathbf{D8)}\ \ \mathsf{ext}\triangleright(a) \ =_{def} \ \mathsf{SUM}(\lambda x[\triangleright\mathsf{P}(x, \mathsf{ext}(a))])$$
$$\mathbf{D9)}\ \ \mathsf{ext}\triangleleft(a) \ =_{def} \ \mathsf{SUM}(\lambda x[\triangleleft\mathsf{P}(x, \mathsf{ext}(a))])$$

Informally, $\mathsf{ext}\triangleright(a)$ is defined to be the sum of those regions which are part of the extension of a during some open time interval immediately following the actual time. Wherever the extension of a is continuous, the equality $\mathsf{ext}(a) = \mathsf{ext}\triangleright(a) = \mathsf{ext}\triangleleft(a)$ must hold; but where a's extension is discontinuous, $\mathsf{ext}\triangleright(a)$ is the extrapolation of its the extensions in the immediate future, whereas $\mathsf{ext}\triangleleft(a)$ is the extrapolation of its extensions in the immediate past.

The extrapolated extension functions allow us to characterise spatial events, which happen between the destruction and creation of chunks. For example, the relation $\mathsf{BreakOff}(a, b)$, which holds at time t just in case a and b are chunks of rigid matter such that a breaks away from b at time t can be defined as follows:

$$\mathbf{D10)}\ \ \mathsf{BreakOff}(a, b) \ \equiv_{def} \ \triangleright\textsc{chunk-of}(\mathbf{rm})[a] \wedge \triangleleft\,\textsc{chunk-of}(\mathbf{rm})[b]) \wedge \mathsf{PP}(\mathsf{ext}\triangleright(a), \mathsf{ext}\triangleleft(b))$$

3.3 Constituent Parts of Rigid Matter

An important property of rigid matter is that the constituent parts of rigid chunks maintain a constant location relative to the boundaries of the chunk. Hence the extension of a 'rigid sub-chunk' (RSC) which is part of a rigid chunk can be traced through time by reference to the extension of the chunk within which it subsists. To fix the relative spatial relationship between the sub-chunk and its parent chunk, we use the 'congruent pairs' relation introduced in (Bennett, Cohn, Torrini and Hazarika 2000a).

$$\begin{aligned}\mathbf{D11)}\ \ \textsc{rsc}[s] \ \equiv_{def} \ & \neg(\mathsf{ext}(s) = \emptyset) \wedge \\ & (\exists \textsc{chunk-of}(\mathbf{rm}), c)(\exists r_s, r_c)[\\ & \quad \mathsf{ext}(s) = r_s \wedge \mathsf{ext}(c) = r_c \wedge \mathsf{P}(r_s, r_c) \\ & \quad \wedge \ \Box((\mathsf{ext}(c) = \emptyset \wedge \mathsf{ext}(s) = \emptyset) \vee \\ & \quad\quad \mathsf{CGpairs}(r_s, r_c, \mathsf{ext}(s), \mathsf{ext}(c)))]\end{aligned}$$

I now define the count noun RMP, standing for 'rigid matter part'. These entities are portions of rigid matter that persist through breakup events unless the actual matter portion of the RMP is broken. For example, figure 1 indicates two RMPs which subsist within a brick that is broken on a wall. After the impact `Part2` becomes co-extensive with one of the chunks formed by the breakup.

Between breakup and merger events an RMP is always co-extensive with some RSC. By referring to the limiting extensions of RSCs at these events we can identify RSCs before and after such an event which share the

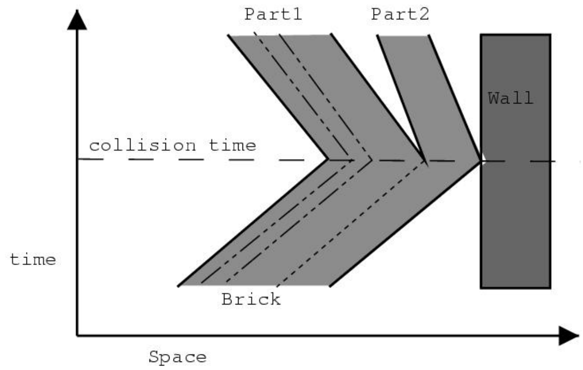

Figure 1: RMPs subsisting within a brick which is broken against a wall.

same limiting extension. These RSCs are both *component slices* of the same RMP, whose existence persists through the event. I define the more general relation of one object being a slice of another as follows:[4]

D12) $\mathsf{Slice}(s, a) \;\equiv_{def}\; \mathsf{TCON}(s) \wedge \Box(\mathsf{ext}(s) = \emptyset \vee \mathsf{ext}(s) = \mathsf{ext}(a))$

In order to facilitate the somewhat subtle definition of an RMP, I introduce the auxiliary relations $\mathsf{FCRSC}\triangleright(r, p)$ and $\mathsf{FCRSC}\triangleleft(r, p)$. These can be read as 'region r faces a component rigid sub-chunk of p in the immediate future/past'. In other words $\mathsf{FCRSC}\triangleright(r, p)$ and $\mathsf{FCRSC}\triangleleft(r, p)$ are true if in the immediate future/past (i.e. during some open interval following/preceding the actual time) there is an RSC which is a component slice of p and whose limiting extension at the actual time is r. Both predicates are also true if r lies within a component slice of p.

D13) $\mathsf{FCRSC}\triangleright(r, p) \;\equiv_{def}\; (\exists^* \textsc{rsc}, s)[\mathsf{ext}\triangleright(s) = r \wedge \mathsf{Slice}(s, p)]$

D14) $\mathsf{FCRSC}\triangleleft(r, p) \;\equiv_{def}\; (\exists^* \textsc{rsc}, s)[\mathsf{ext}\triangleleft(s) = r \wedge \mathsf{Slice}(s, p)]$

One will note that in the definition of FCRSC I have used a quantifier $\exists^*$ instead of the ordinary (sorted) form of quantification used in $\mathfrak{O}$. This is because, usually in that language, when one quantifies over objects of a given kind, one quantifies only over those that exist at the actual time in the actual history. In most cases this is the most useful and intuitive way to quantify; however, for the purpose of certain ontological definitions, such as that of FCRSC, it is much more convenient to quantify directly over all objects of a particular kind, whether or not they exist at the present. Hence, I define

D15) $(\exists^* \kappa, x)[\varphi] \;\equiv_{def}\; \exists t, t'[\mathsf{AT}(t) \wedge \mathsf{Holds\text{-}At}((\exists \kappa, y)[\mathsf{Holds\text{-}At}(\varphi, t)], t')]$

D16) $(\forall^* \kappa, x)[\varphi] \;\equiv_{def}\; \neg(\exists^* \kappa, x)[\neg\varphi]$

Now we come to the RMP definition. The idea is that whenever the extension of an RMP is non-empty it must face an RSC in both future and past directions. Moreover if a region faces in one direction an RSC which is a slice of an RMP, then any RSC which it faces in the other direction must also be a slice of that same RMP.

D17) $\textsc{rmp}[p] \;\equiv_{def}\; \neg(\mathsf{ext}(p) = \emptyset) \wedge \mathsf{TCON}(p) \wedge \Box(\exists r)[\mathsf{ext}(p) = r \wedge (r = \emptyset \vee (\mathsf{FCRSC}\triangleleft(r, p) \wedge \mathsf{FCRSC}\triangleright(r, p))) \wedge (\forall r')[(\mathsf{FCRSC}\triangleleft(r, p) \wedge (\exists^* \textsc{rsc}, s)[\mathsf{ext}\triangleright(s) = r']) \rightarrow (r = r' \wedge \mathsf{FCRSC}\triangleright(r, p))] \wedge (\forall r')[(\mathsf{FCRSC}\triangleright(r, p) \wedge (\exists^* \textsc{rsc}, s)[\mathsf{ext}\triangleleft(s) = r']) \rightarrow (r = r' \wedge \mathsf{FCRSC}\triangleleft(r, p))]]$

Although their definitions specify exactly what conditions an object must satisfy to be an RSC or an RMP, they do not actually guarantee the existence of any such objects. However, since these objects are supervenient on the distribution of the rigid matter in the universe (as represented by the matter type **rm**), suitable existential axioms can easily be given:

A3) $(\forall \textsc{chunk-of}(\mathbf{rm}), c)(\forall r)[\mathsf{P}(r, \mathsf{ext}(c)) \rightarrow (\exists \textsc{rsc}, s)[\mathsf{ext}(s) = r]$

A4) $(\forall \textsc{rsc}, s)(\exists \textsc{rmp}, r)[r \doteq s]$

3.4 Pieces

Now that we have defined RMP we can define several important relationships between chunks and *pieces* of chunks. Unfortunately there seem to be a family of rather different relations corresponding to different senses of the natural language relation 'x is a piece of y'. For instance it is debatable whether x should be taken to be already detached or merely detachable from y; or perhaps x can actually be an integral part of y. In addition, the temporal structure of the 'piece' relation is also subtle. In some contexts the term y refers to an object which exists at the same time as x, while in others it seems to refer to an object which existed prior to the loss of the piece x.

The analysis I have given is adequate to characterise any of these alternatives. To me the following definition seems natural:

D18) $\mathsf{Piece}(a, b) \;\equiv_{def}\; \textsc{rmp}[a] \wedge \textsc{chunk-of}(\mathbf{rm})(a) \wedge \mathbf{P}(\textsc{chunk-of}(\mathbf{rm})[b] \wedge \mathsf{P}(\mathsf{ext}(a), \mathsf{ext}(b)))$

[4] $\mathsf{TCON}(s)$ means that s exists during a connected temporal interval — see (Bennett 2001c).

Here, a is required to be a detached RMP, which was previously part of the rigid chunk b.

4 Identification and Identity of Physical Objects

In specifying a formal ontology, for each type of object we consider we are principally concerned with defining two (closely related) kinds of criterion: a) *identification* criteria, which determine the conditions under which at a particular time a given physical extension is the extension of an object of that kind; and b) *identity* criteria which enable us to say when two observations of an object of a given kind are observations of the same object.

Later I shall use the theory of chunks and pieces to define identity criteria for objects over time that allows objects to persist through events where they lose inessential pieces. However I shall first clarify a number of issues that need to be tackled.

4.1 Vagueness in Identification and Identity

The logic $\mathfrak{O}$ treats objects as having built-in precise identification and identity criteria derived from their sort. Thus, insofar as there is vagueness associated with the extension and identity criteria of an object, this must be regarded as residing within the sort by which the object is individuated, rather than independently in the object itself. For example the extension and identity criteria of a particular cup are vague just because the concept of cup is itself vague. Thus the logical form of a claim that the region r is the extension of 'this cup' can be represented by $\mathbf{S}[\mathsf{ext}(\mathsf{this}(\textsc{cup})) = r]$.[5] Here the $\mathbf{S}$ modality can be read as 'in some sense' (see (Bennett 1998)). It means that there is some *precisification* of the language under which the extension of $\mathsf{this}(\textsc{cup})$ is precisely r.

In most cases it is uncontroversial whether two observations of objects at different times should be said either to be observations of the same or of different thing. However, there will also be occasions where the identity of the objects is profoundly controversial. For instance if a cup were broken in such a way that a piece of the cup could still function as a makeshift cup we might or might not be inclined to say that this piece were in some sense the same cup (albeit damaged) as the original. Our willingness to ascribe this identity would depend on the particular details of the breakage and there could be many borderline cases.

[5] Here 'this' is an ostensive operator which given a sort and evaluated in a particular context (e.g. a context where a speaker is pointing in some definite direction) returns an object of that sort. If you don't like this formalisation of ostension, you could replace $\mathsf{this}(\textsc{cup})$ with a definite description containing the sort CUP.

4.2 Loss of Tiny Parts

One of the most difficult obstacles to formulating reasonable identity criteria for physical objects is to take account of the possibility of objects losing tiny insignificant parts. For example, if a brick is dropped or scraped with a spade, small particles of brick dust will be removed. However, everyday intuition tells us that the original brick still exists, albeit in a slightly damaged form. The big question is, how large and how many pieces can an object lose before it ceases to exist. Informally one might say something like: an object persists as long as it does not lose any 'significant' part. But of course the notion of what is 'significant' is intrinsically vague. In the following sections I shall attempt to give some formal machinery that gives us some purchase on what can be meant by a significant part. I suggest that there are actually a number of different ways in which significance can be quantified and these are in fact relevant to the wide vocabulary of natural language descriptions of objects as being 'worn', 'damaged', 'broken' etc..

4.3 Identity Depends on History

A further complication is that the question of whether an object at some time t (in some possible history) should be regarded as the same thing as some precursor object at time t' depends not only on the states of the object(s) at t and t' but also on the history of changes that occur between t' and t. For instance if I take a long stick (say 1m) and chop it into 5cm pieces, then it does not seem reasonable to take any of these pieces to be the same stick as the original. However, if I push the stick into a power tool that shaves off thin sectional slices of the stick from one end until end up with a remaining stub 5cm long, then there do seem to be grounds for saying that this stub is in some sense the same thing as the original long stick. The rational would be that the continuity of the sticks existence is preserved throughout a series of small changes in its extent but not through the sudden loss of a 'significant' part (or parts).

5 Three Modes of Significance

I have argued that identity criteria for the objects of commonsense reasoning depend essentially on certain intuitions about what parts of an object can be neglected as insignificant. Hence in this section I shall try to formalise some of these intuitions.

The concept of a 'significant part' is extremely slippery. At least three rather different notions of significance appear to be important:

1. A part may be significant purely in virtue of its physical extension. That is it may be too small to consider important or even to small to observe.
2. A part of an object may be insignificant relative to the geometry of the extension of that object — difference between the geometry and topology of the object with and without the part is in some sense 'negligible'.
3. A part of an object may be insignificant in relation to the purpose or functionality of that object.

5.1 Absolute Measures of Size

The most obvious criterion by which a piece of matter might be judged significant is its absolute physical size. Conversely, if a piece is smaller than some particular threshold we may call it insignificant. Actually there are several ways in which we could measure the 'size' of an object including:

1. The total volume occupied by the object.
2. The maximal distance between two points in the object.
3. The diameter of the object's minimal bounding sphere.
4. The diameter of the largest sphere which is wholly within the extension of the object.

In each case a threshold of significance can be arbitrarily defined by reference to some standard sphere, which I denote by **i**, which is taken to have the largest diameter of any sphere that is considered to be 'insignificant'. In other words, any sphere larger than **i** is considered to be significant. Clearly **i** satisfies:

A5) $\mathsf{S}(\mathbf{i})$

A relation, $x >_v y$, which is true when region x has a volume greater than y is 1st-order definable in **RBG**. This seems to require a rather complex definition, so in the current paper I only give a sketch of how it can be done. We note that the volume of x is greater than or equal to y iff: there is some set $R = \{\ldots r_i \ldots\}$ of non-overlapping sub-regions of x for which there is a one-to-one mapping to a set $S = \{\ldots s_i \ldots\}$ (this can contain overlapping regions), such that for each i we have $\mathsf{CG}(r_i, s_i)$ and y is a part of the sum of all the regions in S. Here, CG is the congruence relation (which is definable from P and S). Additional formal machinery is needed to specify the one-to-one mapping. One method is to restrict R and S to sets of tetrahedra each of which can be specified by four spheres centred on its corners. A set of tetrahedra can then be identified with a *scalene sum of spheres* — i.e. a region made up a sum of disjoint spheres all of different sizes (see (Borgo et al. 1996, Bennett et al. 2000a, Bennett et al. 2000c)). The required mapping between the tetrahedra can then be defined in terms of the mapping between pairs of congruent spheres in two scalene sums of spheres.

Once we have the relation $>_v$, the property of a region's having a significant volume is easily defined:

D19) $\mathsf{SigVol}(x) \ \equiv_{def} \ x >_v \mathbf{i}$

If a region contains two points separated by a distance greater than the diameter of **i** it satisfies the SigLen predicate defined by:

D20) $\mathsf{SigLen}(x) \ \equiv_{def} \ \exists y \exists z[\mathsf{P}(y,x) \wedge \mathsf{P}(z,x) \wedge \forall y' \forall z'[(\mathsf{P}(y',y) \wedge \mathsf{P}(z',z)) \rightarrow \neg\exists s[\mathsf{CG}(s,\mathbf{i}) \wedge \mathsf{P}(y'+z',s)]]]$

Finally $\mathsf{SigBS}(x)$ specifies that x can only be bounded by a sphere which is 'significant'; and $\mathsf{SigIS}(x)$ says that x contains a 'significant' sphere:

D21) $\mathsf{SigBS}(x) \ \equiv_{def} \ \neg\exists s[\mathsf{CG}(s,\mathbf{i}) \wedge \mathsf{P}(x,s)]$
D22) $\mathsf{SigIS}(x) \ \equiv_{def} \ \exists s[\mathsf{CG}(s,\mathbf{i}) \wedge \mathsf{NTPP}(s,x)]$

Given that the size of the **i** is arbitrary, it may seem that this does not give use a concrete handle on what is significant. However, from the perspective of supervaluationistic logic, we can regard each choice of diameter for **i** as a precisification of a vague language containing a set of vague predicates for describing the significance of objects. Moreover, since all these concepts are defined by reference to **i**, each such precisification enforces strong logical consistency requirements among the vague concepts.

Many useful concepts for characterising vague spatial relationships are definable in terms of **i**. A relation of *approximate equality* between regions can be defined as follows:

D23) $(x \sim y) \ \equiv_{def} \ \neg\exists s[\mathsf{CG}(s,\mathbf{i}) \wedge ((\mathsf{P}(s,x) \wedge \mathsf{DR}(s,y)) \vee (\mathsf{P}(s,y) \wedge \mathsf{DR}(s,x)))]$

Informally this means it is not possible to insert a 'significant' region in the space between the boundaries of regions x and y. This definition has the shortcoming that hair-like extrusions or intrusions which are too fine to contain a 'significant' region, no matter how long they are cannot make any significant difference between x and y. A better definition might be achieved by also limiting the maximum allowed distance between any part of one region and the nearest part of the other.

I also define the count noun Particle as follows:

D24) $\mathsf{Particle}[p] \equiv_{def}$ CHUNK-OF$(\mathbf{rm})[p] \wedge \exists s[\mathsf{P}(\mathsf{ext}(p), s) \wedge \mathsf{CG}(s, \mathbf{i})]$

So a particle is a chunk that is small enough to be contained within a sphere that is congruent to **i**. Particles represent a type of insignificant object that is very useful for defining certain identity criteria and also events such as abrasion.

The concept of 'weak contact' was proposed by (Asher and Vieu 1995) to model the contact between distinct but touching physical objects. Figure 2 illustrates a weak contact seen at high magnification. In terms of **i** it is straightforward to model that kind of connection holding between objects such that their surfaces are close enough together that no significant region can be interposed between them.

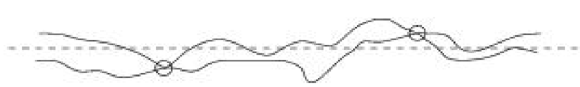

Figure 2: A 'weak' contact at high magnification

5.1.1 i as a Supervaluationistic Parameter

Since the diameter of the maximal insignificant sphere **i** is not fixed relative to the geometrical primitives of $\mathfrak{O}$ it may be regarded as a parameter each of whose values corresponds to a possible precise sense of insignificant — i.e. it specifies a *precisification* of this concept. Since a range of concepts relating to spatial vagueness can be defined from this single vague primitive, one can ensure that all these concepts, while not fixed absolutely, do fit together in a coherent way. Moreover, logical inferences that follow from these definitions, independently of the actual magnitude of **i** can be seen as truly valid despite the underlying vagueness.

This means that a spatial theory including the **i** primitive functions in much the same as a *supervaluationistic* logic such as proposed by (Fine 1975) and investigated in relation to KR applications in (Bennett 1998). Given the potential association between values of **i** and *precisifications* it would be easy to enrich the logic by adding modal vagueness operators which enable one to refer to other possible values of **i**. Thus, using the notation of (Bennett 1998), a formula of the form $\mathbf{S}[\Phi(\mathbf{i})]$ means that $\Phi(\mathbf{i})]$ is true 'in some sense' — i.e. for some value of **i**. Use of such an operator enables one to combine without danger of introducing logical inconsistency, different pieces of information which might employ different senses of significance.

5.2 Minimal Axioms for 'Significant Part'

A second important mode in which an object might be considered significant is in terms of the relative significance of its extension in comparison with the extension of some larger region. Thus I axiomatise the relation of *significant parthood* holding between two regions.

A6) $\mathsf{SP}(r, s) \rightarrow \mathsf{P}(r, s)$
A7) $\mathsf{SP}(r, r)$
A8) $(\mathsf{SP}(r, s) \wedge \mathsf{P}(x, s) \wedge \mathsf{P}(r, x)) \rightarrow \mathsf{SP}(x, s)$
A9) $(\mathsf{SP}(r, s) \wedge \mathsf{CGpairs}(r, s, r', s')) \rightarrow \mathsf{SP}(r', s')$

A6 and **A7** simply ensure that a significant part is always a part and that every region is a significant part of itself. **A8** requires that if r is a significant part of s then any part of s which includes r is also a significant part of s. The final axiom ensures that whether r is a significant part of s depends only on the relative location of r with respect to s. It can also be argued that the SP relation should depend only on the relative size and location of the two regions involved. This can be axiomatised by requiring that the pairs r, s and r', s' are merely *similar* rather than congruent:

A10) $(\mathsf{SP}(r, s) \wedge \mathsf{SimPairs}(r, s, r', s')) \rightarrow \mathsf{SP}(r', s')$

5.3 Other Possible Axioms

I considered adding a further axiom

$$(\mathsf{SP}(r, s) \wedge \mathsf{P}(x, s) \wedge \mathsf{P}(r, x)) \rightarrow \mathsf{SP}(r, x)\ .$$

This claims that if r is a significant part of s then r is also a significant part of any sub-region s' of s of which it is a part (a sort of 'downward' persistence of significance'). However this is not in fact tenable, since it may be possible to find an s' which, although it contains r, does not contain some other part of s that is relevant to the significance of r. For instance figure 3 shows a region $r = a{+}b$ which is the sum of two discs (a and b) that slightly overlap. Here the region of overlap, x, is intuitively significant; but this region may not be considered a significant part of either of the discs.

This example suggests that one might want an axiom specifically guaranteeing the 'significance' of regions

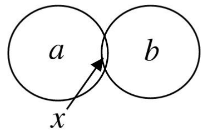

Figure 3: A significant connecting piece.

which are necessary to connect other significant parts of a region. To this end one might add:

$$\textbf{A11)}\ \exists x \exists y[\mathsf{SP}(x, s-r) \wedge \mathsf{SP}(y, s-r) \wedge \exists c[\mathsf{ICP}(c,s) \wedge \mathsf{P}(x,c) \wedge \mathsf{P}(y,c)] \wedge \neg\exists c[\mathsf{ICP}(c, s-r) \wedge \mathsf{P}(x,c) \wedge \mathsf{P}(y,c)]] \rightarrow \mathsf{SP}(r,s)$$

Where, $\mathsf{ICP}(c,s)$ is defined to mean that c is an interior-connected part of s.

Given the vagueness of SP one might wish to go further and axiomatise additional properties of different senses of the concept using a supervaluationistic logic. However for most purposes this treatment will not be required. One can argue that an appropriate degree of vagueness is captured simply by letting SP float free of precise semantics of $\mathfrak{O}$, subject to its being constrained by the above axioms. Moreover, this can be justified in terms of supervaluationistic logic on the same basis as was explained for the case of the **i** constant.

5.4 Identity Sufficient Parts

Finally I define two notions of significant parthood which relate RMPs to the objects in which they subsist. These complement the purely spatial significance relations defined above and enable us to describe significance in terms of higher level properties of objects.

I define the relationship $\mathsf{ISP}(p, o)$ of an RMP being an *identity sustaining piece* of an object. This relation holds whenever p is a piece of o which is sufficient to carry the identity of object o in the event of all other pieces being detached or destroyed.

$$\textbf{D25)}\ \mathsf{ISP}(p,o) \equiv_{def} \textsc{rmp}[p] \wedge \Box(\mathsf{P}(\mathsf{ext}(p), \mathsf{ext}(o)))$$

A stronger relation of *identity necessary piece* can also be defined:

$$\textbf{D26)}\ \mathsf{INP}(p,o) \equiv_{def} \textsc{rmp}[p] \wedge \Box(\mathsf{P}(\mathsf{ext}(p), \mathsf{ext}(o)) \ \Box((\mathsf{ext}(p) = \emptyset \rightarrow \mathsf{ext}(o) = \emptyset))$$

These relations can be used to define identity criteria for artifacts, where identity may not be reducible to questions of the significance of spatial extents, but rather is dependent on specific parts of the object.

We might want to add additional axioms constraining ISPs and/or INPs. For instance, unless all ISPs are required to have a common intersection it would be possible for an object to split into two parts each capable of sustaining its identity and this could be problematical.[6]

6 Object Types and MRPO's

We now come to that part of the ontology where we relate natural physical objects to RMPs using the various vague significance criteria. However, in order to facilitate this I introduce yet one more sort of a theoretical nature.

The vast majority of sort terms used in natural languages refer to either human artifacts, biological organisms (and organs) or geographic features. However, the identification and identity criteria of such objects are affected by a host of issues of conceptual vagueness, which obscure more even more fundamental issues.

In order to carry through my programme of establishing an ontology bottom up (starting from just the spatio-temporal distribution of matter types), I shall start by considering a generic physical object which is devoid of biological or artifactual significance. This will be called a *mesoscopic rigid physical object* (of matter type **m**) and its type formally represented by the sortal MRPO(**m**). As the prototype of such an object may consider the sort 'rock' which denotes a portion of rigid matter with no particular structure or function. Actually 'rock' is slightly more specific than MRPO since it limits the type of rigid matter to various mineral substances and it also suggests that the material is not shaped for any purpose. The sort MRPO is neutral w.r.t. these possible limitations so that it includes rocks, lumps of wood and cups MRPO.[7]

The formal definition of MRPOs is conceptually straightforward but technically complex. The general idea is that an MRPO of matter type **m** is composed of a sequence of slices each of which is a 'significant' chunk of matter type **m**. Moreover, at the time points between these slices, the difference between the limiting extensions of neighbouring component chunks are

[6] Consider a broom which has first its handle then its brush replaced. Maybe we would like both handle and brush to be ISPs? But then what if we separate these parts and join a new brush to the handle and a new handle to the brush?

[7] Consideration of biological and geographical objects would require ontological analysis which is beyond the scope of the present work. But work on formalising these domains can be found in (Cohn 2001, Bennett 2001a, Bennett 2001d).

negligible. Thus at these time points the extensions of MRPOs change discontinuously due to the gain or loss of 'insignificant' parts. (However, in so far as these parts are insignificant the change may be regarded as continuous relative to a certain commonsense abstraction of the material world.) At a point of discontinuity, the extension of the MRPO is taken to be the sum of the limiting extensions of the neighbouring chunks.[8]

I first define 'significant chunk of **m**:

D27) $\textsc{s-chunk}(\mathbf{m})[x] \ \equiv_{def} \ \textsc{chunk-of}(\mathbf{m})[x] \wedge \mathsf{SigIS}(\mathsf{ext}(x))$

D28) $\textsc{mrpo}(\mathbf{m})[p] \ \equiv_{def} \ \neg(\mathsf{ext}(p) = \emptyset) \wedge$
$$\begin{aligned}&\Box((\mathsf{ext}(p) = \emptyset) \vee \\ &\quad (\exists \textsc{s-chunk}(\mathbf{m}), c)[p \doteq c] \vee \\ &\quad (\exists^* \textsc{s-chunk}(\mathbf{m}), c_1)(\exists^* \textsc{s-chunk}(\mathbf{m}), c_2) \\ &\qquad [\mathsf{ext}\triangleleft(c_1) \sim \mathsf{ext}\triangleright(c_2) \wedge \\ &\qquad\quad \mathsf{ext}(p) = (\mathsf{ext}\triangleleft(c_1) + \mathsf{ext}\triangleright(c_2))]) \\ &\wedge \\ &\Box((\exists^* \textsc{s-chunk}(\mathbf{m}), c)[\mathsf{ext}\triangleleft(p) \sim \mathsf{ext}\triangleright(c)] \rightarrow \\ &\quad ((\mathsf{ext}(p) = (\mathsf{ext}\triangleleft(p) + \mathsf{ext}\triangleright(c))) \wedge \ \triangleright(p \doteq c))) \\ &\wedge \\ &\Box((\exists^* \textsc{s-chunk}(\mathbf{m}), c)[\mathsf{ext}\triangleleft(c) \sim \mathsf{ext}\triangleright(p)] \rightarrow \\ &\quad ((\mathsf{ext}(p) = (\mathsf{ext}\triangleleft(c) + \mathsf{ext}\triangleright(p))) \wedge \ \triangleleft(p \doteq c)))\end{aligned}$$

To ensure that every significant chunk of rigid matter corresponds to a slice of some MRPO we have the following axiom:

A12) $(\mathsf{RigidMatter}(\mathbf{m}) \wedge \textsc{chunk-of}(\mathbf{m})[c]) \rightarrow (\exists \textsc{mrpo}(\mathbf{m}, x)[x \doteq c]$

6.1 Identity Criteria for Artifacts

Although, artifacts are co-extensive with purely physical bodies (which can be modelled as MRPOs), their identity and identification criteria are far more subtle.

In respect of its identification criteria, an artifact of a given kind generally has some function which it must fulfill or be capable of fulfilling in order to count as falling under that kind. For instance a CUP might be defined as 'a rigid body that has a capacity to hold liquid and (in its usual sense) can be held easily in one hand'. Containment of liquid has been addressed by (Hayes 1985) and (Randell et al. 1992).

While such properties and their roles in supplying identity criteria are undoubtedly extremely complex there seems to be no reason why they could not be articulated within $\mathfrak{O}$ in terms of relations such as ISP. Moreover, the criteria of identity through time could be specified by means of count noun definitions similar to that of MRPO but allowing for possible loss (or gain) of parts which, though noticeable (i.e. 'larger' than **i**), do not result in the loss of essential properties.

Nevertheless, the specific functional properties that are necessary to a particular type of artifact will in many cases be highly controversial. Hence although we may define some formal notion of 'cup' in terms of its enduring capability to contain liquid this may not always (especially if we consider extreme examples) correspond with our intuitive (and possible inconsistent) ideas about cups. Here again, it may be possible to tackle this problem using supervaluation semantics (Fine 1975, Bennett 1998) to model the intrinsic conceptual vagueness found in the meaning of many artifact count nouns.

It is controversial whether the identity (as opposed to identification) criteria of artifacts and biological objects can be reduced the identity criteria of the MRPOs in which they subsist. It seems that this is not always the case and that to get adequate identity criteria we have to have an explicit theory of significant component parts of an artifact (such as suggested in (Simons 1987) and (Cohn 2001)).

7 Wear Damage and Breakage

I now sketch how we could apply the theory of rigid matter portions together with the various criteria of significance to the definition of a variety of kinds of destructive event which can befall a physical object.

A very important concept in the description of faults in physical mechanisms is wear cause by abrasion between two components. When one solid body rubs against another, this is likely to result in the loss of microscopic particles from one or both bodies. At first this will be unnoticeable and will not affect the working of the mechanism; however, if the abrasion continues, eventually it may result in significant distortion of a component which will cause a malfunction of the mechanism.

Abrasion can be modelled quite straightforwardly *via* the definition of particle. However, there are some issues which must be tackled. In particular there is the need to model the fact that a long series of tiny abrasions can eventually result in critical damage or breakage. This can be accommodated within the proposed theory by noting that successive loss of insignificant particles can build up to the loss of a significant part (an SP) and eventually even result in the destruction of all identity sustaining parts (ISPs).

[8] The rationale behind this is that states of contact *dominate* states of disconnection (see Galton (2000)).

Damage which is significant but does not result in the destruction of an object — e.g. the chipping or scratching of a brick — can be characterised as a loss of significant parts while retaining an identity sufficient part.

Breakage on the other hand occurs when an event results in the loss of all identity sufficient parts. As we have seen, the RMP ontology is well suited to describing breakages and enables to identify fragments as being pieces of and object which has been destroyed.

To fully characterise breakage of specific types of artifact we have to axiomatise the ISP relation in terms of whether some RMP is suitable to performing the essential functions of the artifact type. For instance a 'broken cup' might be defined as an item which once had the capacity to fulfill the function of a cup but, due to some damaging event, no-longer has this potential.

Actually there are many subtleties, so we have to be extremely careful in making such stipulations. For instance, if we break a cup we may still find that it can hold fluid within a concavity in one or more of the resulting pieces of cup. To handle this case one might say that a 'broken cup' is a (multi-piece) object made up of one or more pieces all of which were at some time parts (and indeed comprise all significant parts) of a body which was a cup, and such that for all these remaining pieces the capacity to participate in one or more of the essential functions of a cup is significantly reduced. These essential functions will include at least: capacity to hold liquid, facility to drink from, facility to hold.

8 Conclusion

It must be acknowledged that the approach pursued in the current paper is open to criticism on the grounds that its complexity renders it unsuitable for practical applications. My response is that the principal purpose of ontology is to specify the logical structure of propositions about the world in a way which is in some sense 'correct' rather than in a way that is easily manipulable by automated algorithms. Thus ontology should really only define the nature of reasoning problems rather than actually solving them.

Given that we accept this, the separate task of designing reasoning problems could still be tackled in a number of different ways. One would be to look for heuristic theorem proving algorithms that work well in many cases, despite the intractability of the problem in the worst case. Another approach (which I favour) would be to identify systems of concepts that can be reasoned with effectively (e.g. (Renz and Nebel 1999, Wolter and Zakharyaschev 2000)) and thus can be employed within a tractable sub-language of that used to articulate the whole ontology.

One might say that current paper has raised many more problems than it has actually solved. Nevertheless, I believe that I have succeeded in formalising certain concepts that are of fundamental importance in characterising the identity criteria of commonsense physical objects. In this respect the formal framework presented so far may be seen as providing a logical toolkit by means of which a more comprehensive ontological theory could be constructed. Development of such an ontology is the subject of ongoing work.

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