SPATIAL COGNITION AND COMPUTATION, *3*(2&3), 119–136
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Qualitative Extents for Spatio-Temporal Granularity

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The concepts of everywhere and somewhere generate three spatial extents: (1) everywhere, (2) somewhere but not everywhere, and (3) nowhere. These three extents can be used for the granular description of spatial regions. When spatio-temporal regions are considered there are two additional concepts: always and sometimes. The interaction of the four concepts can be used to produce various systems of spatio-temporal extents. The paper shows that if we want to give granular descriptions of spatio-temporal regions, then the system of extents needs to be chosen carefully. A suitable system of eleven spatio-temporal extents is identified, and a simplified system of six extents is also described. The systems of extents presented here are important in that they will allow various structures and theories understood only in a purely spatial context to be generalized to the spatio-temporal case.

Keywords: qualitative reasoning, granularity, spatio-temporal reasoning

1 Introduction

> *Over the last few years there have been some places in the world where the latest advances in spatial information theory have always been the main topic of conversation, and other places where this has never been the case.*

The preceding sentence makes a qualitative statement about the spatio-temporal extent of a region consisting of those locations in space-time where the latest

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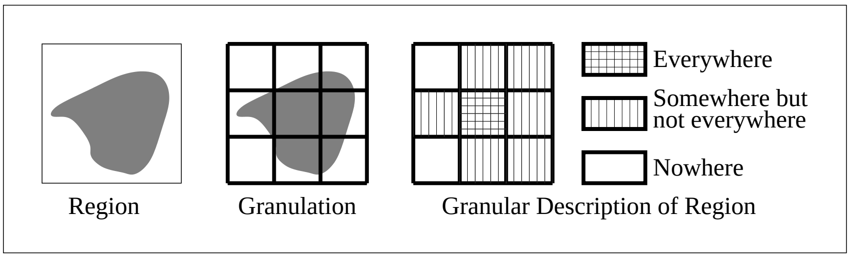


Figure 1. Qualitative spatial extents used for granular description

advances in our subject are the main topic of conversation. As another example of such a statement, incompatible with the previous one, consider the following.

> *Over the last few years there have been some times when the latest advances in spatial information theory have been the main topic of conversation everywhere in the world, and other times when nowhere in the world has this happened.*

Simpler than these spatio-temporal extents are purely spatial ones appearing in statements like *The importance of spatial information is recognized everywhere*, and *Only some parts of the forest are affected by pollution*, and *None of the forest is affected by snow*. These spatial examples exemplify respectively the three qualitative spatial extents:

- *everywhere*
- *somewhere, but not everywhere*
- *nowhere*

These three qualitative spatial extents have been widely used in the approximate or granular description of regions. The general approach is well-known, but it is illustrated in Figure 1 since we need to recall the spatial case before we can describe the generalization to the spatio-temporal. Where a region is part of a space that has been subjected to a granulation (usually a partition of the space) we can describe the region in an approximate way by specifying, for each of the cells in the granulation, the extent to which the region occupies the cell. The three extents used in the simplest case arise from the two concepts *everywhere* and *somewhere*. It should be noted that *somewhere* as used here does not exclude the possibility of *everywhere*. The notations $\Box$ and $\Diamond$ will be used for the concepts *everywhere* and *somewhere* respectively. Figure 2 illustrates how the three extents are generated by the two concepts. On the left of the figure we have the two

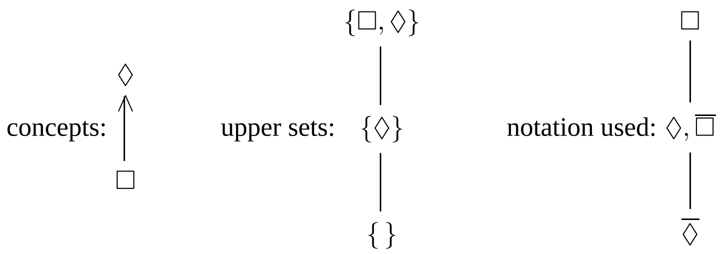


Figure 2. Basic System of Three Spatial Extents

concepts ordered by implication. The ordering expresses the fact that if a region is occupied everywhere then it is occupied somewhere. The ordering restricts the possible combinations of the two concepts which can hold. We can have (1) both $\Box$ and $\Diamond$, (2) only $\Diamond$, or (3) neither. These three extents are the three upper sets[1] of the partially ordered set $\{\Box, \Diamond\}$. The lattice of upper sets appears in the centre of figure 2. It is convenient to introduce a notation for extents which is not that of the upper sets and this is shown on the right of the figure. Here $\overline{\Diamond}$ should be read as the negation of *somewhere* i.e. nowhere, and $\Diamond, \overline{\Box}$ as the conjunction somewhere and not everywhere.

This threefold classification has been used by Worboys (1998b) as the basis of a theory of finite resolution spatial data, and underlies all applications of rough set theory in the spatial context (Beaubouef & Petry, 2001; Bittner & Stell, 2001). The system of three extents is not the only one, and Bittner and Stell (1998) and Bittner (1997) have shown how it can be made more detailed by taking account of how the region being described interacts with the boundaries between each pair of cells in the granulation. There appears to have been little investigation of how a system of spatial extents might be generalized to deal with spatio-temporal regions. This, then, is the question addressed by the present paper: what systems of qualitative extents are there which can be used for the granular description of spatio-temporal regions?

1.1 Structure of the Paper

In Section 2 some related work is discussed. This is divided into two subsections: work on relations between regions, and work on change over time and spatio-temporal data. The relevance of systems of relations, such as RCC5, to the aims of the present paper is outlined in Section 2.1.2. The main technical content of the paper appears in Sections 3, 4, and 5. In Section 3 a first set of qualitative spatio-temporal extents is derived. The suitability of these for making granular

[1] An upper set X of a partially ordered set Y is a subset $X \subseteq Y$ such that $x \in X$ and $y \geq x$ imply $y \in X$.

descriptions is considered in the following section. It emerges that, while a useful set of extents for some purposes, they are unsuitable if we wish to take an already coarsened region and give it a still coarser description. A system of extents which does not suffer from this problem is presented in Section 4.2. A reduced set of only six extents is given in Section 5, and these should be useful when the full system provides a too detailed classification. Finally, conclusions and suggestions for further work are presented in Section 6.

2 Related Work

2.1 Relations Between Regions

Systems of extents for the granular description of regions are closely related to systems of relations between regions. Here we briefly review work on relations between regions before explaining how it is relevant to the present paper.

2.1.1 Work on Systems of Relations

Two spatial regions may relate to each other in a variety of ways, examples being that one region is a proper part of the other or that the two regions are equal. There are several well-known ways of systematizing the possible relations. One family of relations is that provided by the RCC work (Cohn, Bennett, et al., 1997). In this family the simplest classification is provided by the RCC5. More detailed classifications are the RCC8, which takes account of boundaries, and RCC23 which is sensitive to convexity.

Another approach to relationships between pairs of regions, is the work of Egenhofer and Franzosa. Given spatial regions A and B, we can measure how A relates to B by noting which of the intersections between interiors and boundaries of A and B are empty and which are non-empty. This way of describing the relationship of A to B is known as the "4-intersection model", and is studied in Egenhofer and Franzosa (1991). The 4-intersection model is relatively coarse. Finer distinctions between relationships of regions can be obtained by counting the number of connected components in the intersection of A with B, and in certain other regions determined by A and B. The investigation of this use of connected components is due to Galton (1998). Another way of describing the relationship of A to B, which provides for finer distinctions than in Galton's technique, is a refinement of the 4-intersection model also by Egenhofer and Franzosa (1995). Some recent work in this area includes the investigation of relations between three dimensional regions (Zlatanova, 1999). Since it is possible to regard time as the third dimension, this is likely to provide one way of treating relationships between two dimensional spatial regions which vary over a one dimensional time.

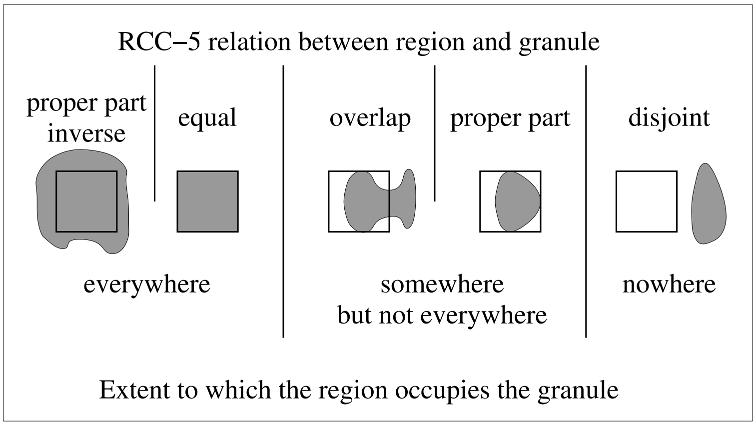


Figure 3. Relation between RCC5 and the 3 basic spatial extents

2.1.2 Relevance of Systems of Relations to Granularity

To describe how systems of relations between regions are relevant to extents, let us assume a space S which has been subjected to a granulation G. We can take G to be a partition of S into cells, or granules, but in fact the basic idea does not depend critically on G being a partition. Given any region r of S, we can describe r with respect to the granulation by giving the relationship between each granule $g \in G$ and the region r. General systems for relations between arbitrary regions are not quite what we need here, because we are only interested in the part of r which intersects g. That is, we are interested in the relationship of the intersection $r \cap g$ to g. Thus the usual RCC-5 relations collapse to just three possibilities, as shown in figure 3. In the case that G forms a partition of S, we can recover the RCC-5 relation between r and g by examining the the extent to which r occupies g' for granules $g' \neq g$. For example, if r occupies everywhere in g, we can distinguish between the two possibilities that the RCC-5 relation is O or PP by determining whether the extent to which r occupies g is always *nowhere* when $g \neq g'$.

In a similar way we can obtain a system of boundary sensitive extents by starting from the RCC8 relations. This tie-up between the widely studied systems of relations, and the as yet less studied notion of extents for granularity, should be useful to the development of both notions. In particular the systems of extents derived later in this paper raise the question of whether they arise from systems of relations between spatio-temporal regions. Answering this question, which is beyond the scope of the present paper, should provide useful information about the possible relationships between spatial regions varying over time.

2.2 Spatio-temporal Granularity

From a conceptual viewpoint, space and time are very closely linked. The use of spatial metaphors in thinking about time has been stressed by Lakoff (1987), and the relevance of this to geographic information systems has been noted by Frank (1998). The collection (Egenhofer & Golledge, 1998) contains much further material on time and its interaction with space from the GIS perspective. Despite the close link between time and space, the qualitative description of spatio-temporal data in a unified way is still far from being understood. In fact there are many difficult issues concerning changes in any kind of data over time, without introducing problems which are specific to spatial data. A unifying model of temporal granularities, independent of spatial issues, is proposed by Bettini, Jajodia, and Wang (2000).

Hornsby and Egenhofer (1997, 2000) have proposed a change description language capable of modelling changes, over time. In other work (Hornsby & Egenhofer, 1999) they have addressed views at various levels of detail of objects which are subject to change. This uses a lattice of levels of detail in a similar way to the stratified map spaces of Stell and Worboys (1998).

Medak (1999) has proposed a formal approach to change over time in spatio-temporal databases. This uses four basic operations affecting object identity: create, destroy, suspend and resume. Algebraic rules for these operations are presented, and an implementation in the functional programming language Haskell is given.

In a theory of spatio-temporal granularity there must be an appropriate analogue of the notion of partition in the purely spatial sense. The work of Erwig and Schneider (1999) has provided a formal model of spatio-temporal partitions.

3 A First Set of Eleven Spatio-Temporal Extents

We have seen in Section 1 that the spatial concepts of *everywhere* and *somewhere* generate a system of three extents which can be used for the granular description of spatial data. In this section we show how to extend this by the addition of time to the purely spatial case.

In the spatio-temporal case there are four concepts which can be used to describe extents: *everywhere*, *always*, *somewhere* and *sometime*. The notation $\blacksquare$ will be used for *always* and $\blacklozenge$ for *sometimes*. Combinations of the four concepts can lead to complex situations, as evidenced by the examples at the very start of this paper. Figure 4 shows the possible compound concepts and the implications between them. Note the distinction between *somewhere always* and *always somewhere*. If we say an event occurs always somewhere, we mean that for each time there is a place where the event occurs, but different times may have different places. For example, the British Empire was once described as ‘the empire on which the Sun never sets’ because there was always somewhere in the empire where the Sun had not set. If we say an event occurs somewhere always, we mean that it happens in some fixed place for every time.

The possible combinations of the six compound concepts which may hold together are determined by the upper sets of the lattice in figure 4. This lattice of upper sets gives us a lattice of eleven spatio-temporal extents and these appear in figure 5. An explanation of the extents together with examples is given in Table 1.

In this table the top and bottom elements of the lattice are omitted as it should be clear what is meant by the extents everywhere always ($\Box\blacksquare$) and never anywhere ($\overline{\Diamond\blacklozenge}$). In the examples, which appear in the right hand column of the table, we have used a simple case of a space shown as a single rectangle at three moments in time. This is indicated by the text on the first example. Although the examples thus use a discrete time and a continuous space, this is merely for the ease of drawing simple examples. The concepts of somewhere and at some time are not tied to any particular kind of space or time and thus the notions of extent are quite general and are valid whether space or time is discrete or continuous. In reading the table it is important to bear in mind that 'everywhere is occupied at some time' can easily be mis-interpreted. It is used to mean that for every element of the space there is some time at which that element is occupied. It does not mean that there is a particular time at which the whole space is simultaneously occupied. The discussion in Section 4.1 may be helpful in interpreting the diagrams on the right hand side of the table.

4 Spatio-Temporal Granulation

4.1 First Coarsening

To illustrate the use of the extents obtained in the previous section, consider the diagram in figure 6. This figure shows one space at nine times, and for each time a region in the space is shown by a shaded area. Taken as a whole we have a single spatio-temporal region. A physical interpretation could be that we are looking at material on a conveyor belt (perhaps clay in a brick making factory). The nine times correspond to nine snapshots of the state of the conveyor. In such a manufacturing example, we might want to use spatio-temporal extents as part of the knowledge representation component of an automated monitoring system. To summarize the spatio-temporal situation we can impose a granulation, which is indicated by the bold lines in figure 7. The granulation has divided the space into nine granules or cells. In this paper it will be assumed that a spatio-temporal granulation is determined by two granulations: one of time and one of space. While more general spatio-temporal granulations are clearly important, this simple kind provides sufficient material for these initial investigations.

We can give a coarse description of the spatio-temporal region by specifying for each granule the extent to which the region occupies it. This is the extension

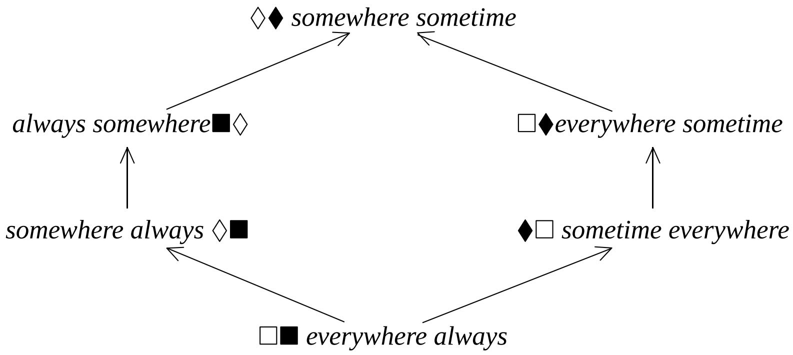


Figure 4. Implications between the six Compound Concepts

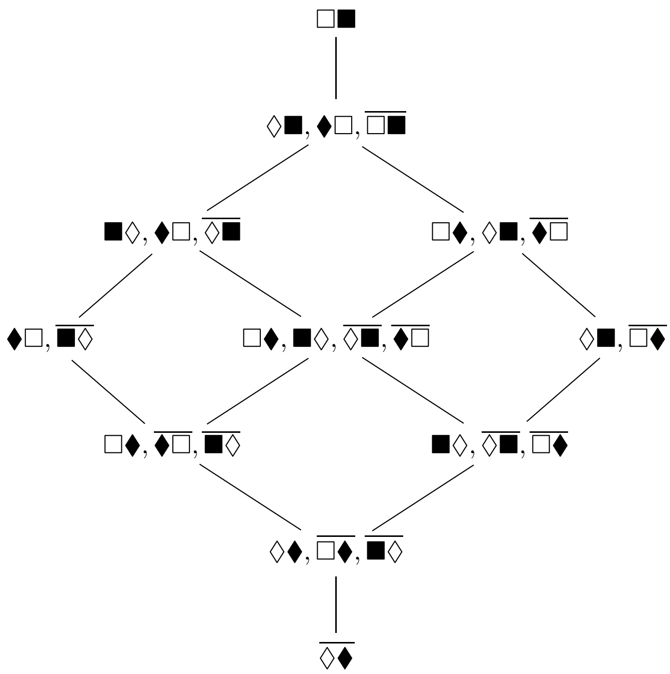
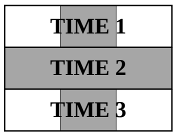
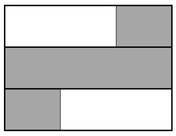
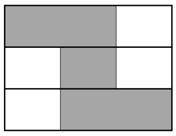
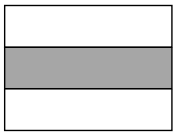
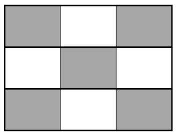
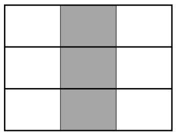
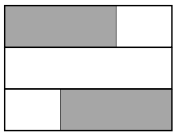
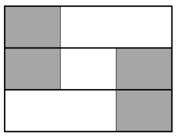


Figure 5. The First System of Spatio-Temporal Extents

Table 1
Spatio-temporal extents and their interpretations

$\Diamond\blacksquare$, $\blacklozenge\Box$, $\overline{\Box\blacksquare}$	Somewhere is always occupied, and at some time everywhere is, but not everywhere is always occupied.	
$\blacksquare\Diamond$, $\blacklozenge\Box$, $\overline{\Diamond\blacksquare}$	At every time some part of the space is occupied, and at some time all of the space is occupied, but there is no part of space which is always occupied	
$\Box\blacklozenge$, $\Diamond\blacksquare$, $\overline{\blacklozenge\Box}$	Everywhere in the space is occupied at some time, and some place is always occupied, but at no time is the entire space occupied.	
$\blacklozenge\Box$, $\overline{\blacksquare\Diamond}$	At some time all the space is occupied, but at some time none of the space is occupied.	
$\Box\blacklozenge$, $\blacksquare\Diamond$, $\overline{\Diamond\blacksquare}$, $\overline{\blacklozenge\Box}$	Everywhere in space is occupied at some time, and at each time somewhere is occupied, but there is not time at which all of space is occupied nor anywhere in space which is always occupied.	
$\Diamond\blacksquare$, $\overline{\blacksquare\Diamond}$	Somewhere in space is occupied for all time, and for some time nowhere is occupied.	
$\Box\blacklozenge$, $\overline{\blacklozenge\Box}$, $\overline{\blacksquare\Diamond}$	Everywhere in space is occupied at some time, but there is no time at which all space is occupied, and at some time nowhere is occupied.	
$\blacksquare\Diamond$, $\overline{\Diamond\blacksquare}$, $\overline{\Box\blacklozenge}$	At every time somewhere is occupied, but there is nowhere which is always occupied, and somewhere is never occupied.	
$\Diamond\blacklozenge$, $\overline{\Box\blacklozenge}$, $\overline{\blacksquare\Diamond}$	At sometime somewhere is occupied, but somewhere is never occupied, and at sometime nowhere is occupied.	

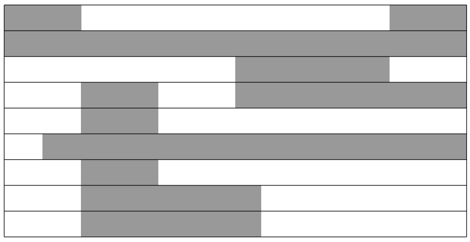


Figure 6. Example, stage 1. One space at nine times

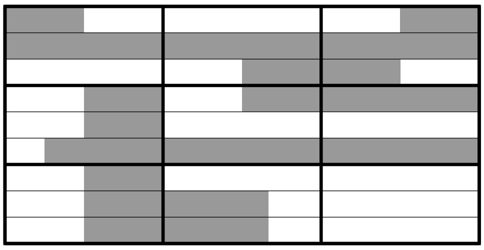


Figure 7. Example, stage 2. First Spatio-Temporal Granulation

to the spatio-temporal case of the simple example which appeared in figure 1. To achieve this for our example, we use the extents from figure 5 to obtain the diagram in figure 8.

4.2 Repeated Coarsening

Now, continuing with the same example, suppose a further granulation is performed. This is indicated by the bold lines in figure 9. Each of these four granules has to be assigned one of the extents. When we consider how to do this we find a difficulty with the interpretations given to some of the eleven extents. In particular consider the large granule in the upper right corner. This granule is made up of two cells. One of these cells is occupied to extent $\blacklozenge\square, \overline{\blacksquare\lozenge}$, and the other to extent $\blacksquare\lozenge, \blacklozenge\square, \overline{\blacklozenge\square}$. If we were coarsening in one step from the space in figure 6, and using the extents in figure 5, the final extent would be $\blacksquare\lozenge, \blacklozenge\square, \overline{\lozenge\blacksquare}$. However, this cannot be the correct way to coarsen the two cells with extents $\blacklozenge\square, \overline{\blacksquare\lozenge}$, and $\blacksquare\lozenge, \blacklozenge\square, \overline{\blacklozenge\square}$. This is because two such cells could equally well have arisen in an alternative way, as shown in figure 10. In the figure each of the arrows represents one coarsening step. In fact we can see that there is no way to fill in the question mark in figure 10 with one of the eleven values from figure 5. The problem arises

$\blacklozenge\square, \overline{\blacksquare\lozenge}$	$\blacklozenge\square, \overline{\blacksquare\lozenge}$	$\blacksquare\lozenge, \blacklozenge\square, \overline{\lozenge\blacksquare}$
$\lozenge\blacksquare, \overline{\blacksquare\lozenge}$	$\blacklozenge\square, \overline{\blacksquare\lozenge}$	$\blacklozenge\square, \overline{\blacksquare\lozenge}$
$\lozenge\blacksquare, \overline{\blacksquare\lozenge}$	$\lozenge\blacklozenge, \overline{\square\blacklozenge}, \overline{\blacksquare\lozenge}$	$\overline{\lozenge\blacklozenge}$

Figure 8. Example, stage 3. Coarse Spatio-Temporal Description

$\blacklozenge\square, \overline{\blacksquare\lozenge}$	$\blacklozenge\square, \overline{\blacksquare\lozenge}$	$\blacksquare\lozenge, \blacklozenge\square, \overline{\lozenge\blacksquare}$
$\lozenge\blacksquare, \overline{\blacksquare\lozenge}$	$\blacklozenge\square, \overline{\blacksquare\lozenge}$	$\blacklozenge\square, \overline{\blacksquare\lozenge}$
$\lozenge\blacksquare, \overline{\blacksquare\lozenge}$	$\lozenge\blacklozenge, \overline{\square\blacklozenge}, \overline{\blacksquare\lozenge}$	$\overline{\lozenge\blacklozenge}$

Figure 9. Example, stage 4. Second Spatio-Temporal Granulation

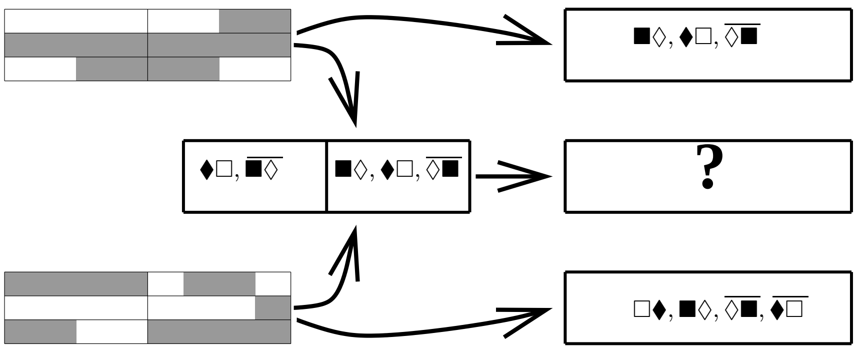


Figure 10. The Problem of Repeated Coarsening

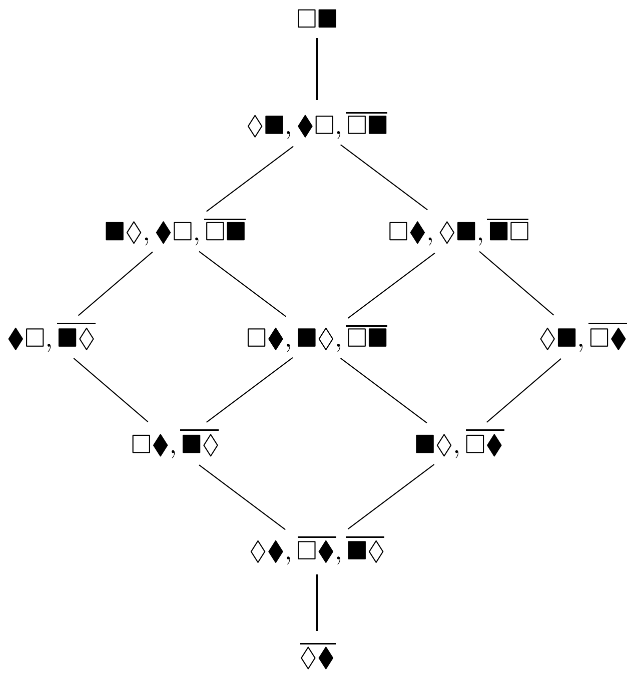


Figure 11. Extents suitable for repeated coarsening

because the values refer to pairwise disjoint possibilities, and the solution is to weaken some of the eleven possibilities. This produces the lattice in figure 11, which differs from the previous one in five places.

The new system of extents no longer describes eleven pairwise disjoint possibilities. For example, if a granule is occupied to extent $\blacklozenge\Box, \overline{\overline{\blacksquare\Diamond}}$, this implies it is also occupied to extent $\Box\blacklozenge, \overline{\overline{\blacksquare\Diamond}}$. These two extents are both needed because in certain circumstances we may have enough information to be sure the extent is $\blacklozenge\Box, \overline{\overline{\blacksquare\Diamond}}$, but in other circumstances we may only be certain of $\Box\blacklozenge, \overline{\overline{\blacksquare\Diamond}}$ without being able to exclude the more informative $\blacklozenge\Box, \overline{\overline{\blacksquare\Diamond}}$. These relationships provide a partial order, $\sqsubseteq$, on the extents with $e_1 \sqsubseteq e_2$ being interpreted as e_1 is less informative than e_2. Among the relationships are $\Box\blacklozenge, \overline{\overline{\blacksquare\Diamond}} \sqsubseteq \blacklozenge\Box, \overline{\overline{\blacksquare\Diamond}}$ and $\blacksquare\Diamond, \overline{\Box\blacklozenge} \sqsubseteq \Diamond\blacksquare, \overline{\Box\blacklozenge}$. The four elements $\Diamond\blacksquare, \blacklozenge\Box, \overline{\overline{\Box\blacksquare}}$, and $\blacksquare\Diamond, \blacklozenge\Box, \overline{\overline{\Box\blacksquare}}$, and $\Box\blacklozenge, \Diamond\blacksquare, \overline{\overline{\blacksquare\Box}}$, and $\Box\blacklozenge, \blacksquare\Diamond, \overline{\Box\blacksquare}$ are also related, in a way easily determined by the implications of figure 4. The remaining three elements are related only to themselves.

It is worth noting that the loss of detail when coarsening in two steps as opposed

Table 2
Rules for determining occupation concepts holding for g

$\mathcal{O}$	Condition for $\mathcal{O}(g)$ to hold
□■	$\forall i, j$ □■(g_{ij})
◊⧫	$\exists i, j$ ◊⧫(g_{ij})
⧫□	$\exists i, j$ ⧫□$(g_{ij}) \wedge \forall k\ k \neq j \Rightarrow$ □■(g_{ik})
◊■	$\exists i, j$ ◊■$(g_{ij}) \wedge \forall k\ k \neq i \Rightarrow$ □■(g_{kj})
■◊	$\forall i \exists j$ ■◊(g_{ij})
□⧫	$\forall j \exists i$ □⧫(g_{ij})

to one was anticipated in the equations proposed by Stell and Worboys (1998) but in that paper we had no explicit example of the phenomenon.

4.3 Rules of Coarsening

We now present the rules which specify how to coarsen a region labelled by the eleven values in figure 11. To describe the rules, we assume a matrix of cells is being condensed into a single granule.

$$\begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{bmatrix}$$

As in the example, each row in the matrix represents the same space; the m different rows corresponding to m different times. Each column represents a part of the space at m different times. In the matrix each g_{ij} is a cell and the notation □■(g_{ij}) will be used to denote that g_{ij} is occupied to the extent □■. The corresponding notation will be used for each of the other six basic concepts. In Table 2 we give the rules for determining when each of the six concepts holds for g. From these basic concepts we can determine which of the eleven extents holds for g, since each extent is a conjunction of these basic concepts and their negations. Using these rules it is now possible to provide the correct labels for the granulation in two stages of the space in figure 6; the result appears in figure 12.

5 Six-Valued Sub-system of Extents

In some applications, the system of eleven extents may be too detailed. Thus it is useful to have the subsystem shown on the right of figure 13. These six ex-

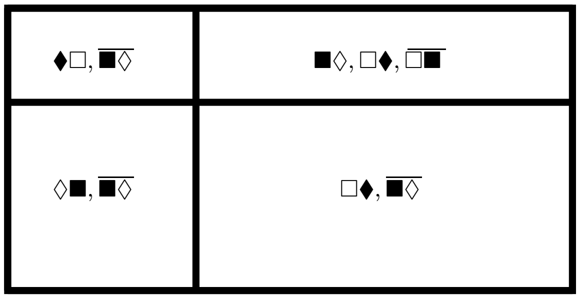


Figure 12. Example, stage 5. Final Spatio-Temporal Granulation

tents arise from the eleven by collapsing each connected component of the partial order $\sqsubseteq$ to its least element. This collapsing process is illustrated in figure 14. Alternatively, the sub-system can be generated from the four concepts shown on the left of figure 13. These four concepts generate a lattice of six jointly exhaustive and pairwise disjoint possibilities.

6 Conclusions and Further Work

This paper has shown how the three notions of spatial extent which arise from the concepts *everywhere* and *somewhere* can be generalized to the spatio-temporal case. This generalization arises from the four concepts *everywhere*, *somewhere*, *always* and *sometimes*. From these four concepts, eleven notions of extent are generated, but we have seen that the most obvious system of eleven extents is not in fact suitable for the granular description of spatio-temporal regions. The reason for this lack of suitability was the failure of the first set of eleven extents to handle the repeated coarsening of regions. A modified system of eleven extents, which are no longer pairwise disjoint, was obtained and the rules which allow these to be used in repeated coarsening were determined. The full system of eleven qualitative spatio-temporal extents may be too detailed for some applications, so a simpler system of only six extents was shown to be obtainable from the eleven. This set of six does have the property of being pairwise disjoint. Although there has been much work on the representation of spatio-temporal change, and some of this has been discussed in Section 2 above, the use of a system of extents which can be used for the granular description of spatio-temporal regions does not seem to have been investigated before.

The work in this paper has been directed towards the identification of a system of extents, and, having accomplished this, there is much further work for which it

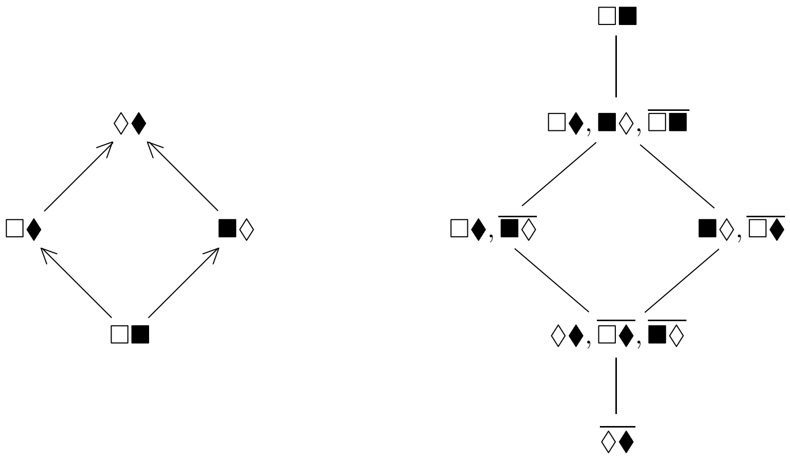


Figure 13. The Six Spatio-Temporal Extents

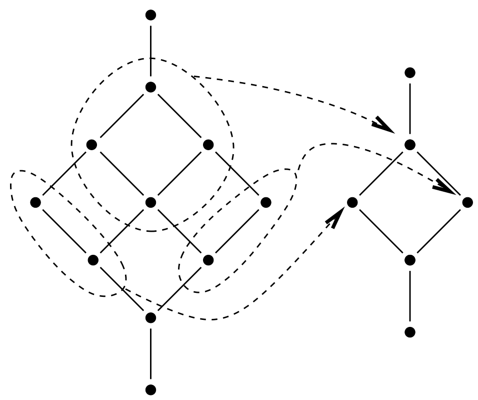


Figure 14. Collapsing the eleven extents to six

provides a foundation. Since the system of extents is a generalization of the tripartite classification used in Worboys' work on finite resolution spatial data (1998b), it would be fruitful to investigate how far Worboys' results can be generalized to spatio-temporal data by using the extents derived in this paper.

It should be possible to extend the system of eleven spatio-temporal extents to a more detailed classification by taking into account the boundaries of the cells in the granulation. This would form a natural generalization to the spatio-temporal context of the researches of Bittner and Stell (Bittner, 1997; Bittner & Stell, 1998). A related question is how relations between granularly represented spatio-temporal regions can be described. What, for example, are the appropriate analogues of systems such as RCC5, RCC8, and the 4- and 9- intersection models? One direction to investigate this would be by using the systems of spatio-temporal extents to generalize the work in (Bittner & Stell, 2001) beyond the purely spatial case.

Time has been assumed to be one dimensional in generating the system of extents. However, models of time with two dimensions (event time and database time) are of practical significance (Worboys, 1998a). By taking time to have two dimensions we would have two notions of always and two notions of sometimes. When these are combined with the two spatial concepts of everywhere and somewhere the resulting system of extents would clearly be more elaborate just eleven cases, but further work would be needed to determine its exact form, and to find appropriate sub-systems for particular applications. Other work on granularity for a structured notion of time is reported in (Stell, 2003).

Acknowledgement

This work was supported by an EPSRC grant "Digital Topology and Geometry: An Axiomatic Approach with Applications to GIS and Spatial Reasoning".

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