Granularity in Change over Time

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1 Introduction

1.1 Evolution over Time

We can describe the world, at any one time, as consisting of entities which may possess attributes, such as colour, spatial location, age, and so on. When we consider two times we find relationships between the entities at one time and those at another. For example, a piece of land may be divided into two, the two parts thus being related to the original one; a child is related to its parents; two entities at different times may be regarded as being "the same person"; what was once divided may coalesce into a single entity at a later time. These thoughts lead to a view of entities evolving over time which suggests the picture of figure 1.

The informal idea of entities evolving or being related to other entities, or being regarded as the same entities at different times, appears natural, and formally this looks like a relation. To examine what properties the relation has, consider the specific example illustrated in figure 2. Here we have a portion of land at three times. At the first time the land is divided into two separate fields, A and B. By time 2, these two have been amalgamated into a single field, C, which is later subdivided in a different way to give fields D and E. Overall there are five entities A, B, C, D, E, and in making a theory of change over time it is necessary to identify what relations exist between the entities at the various times.

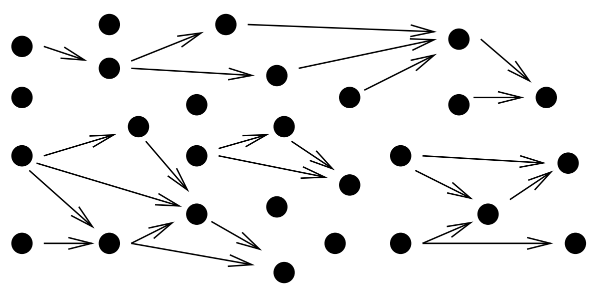


Figure 1: Informal Idea of Evolution

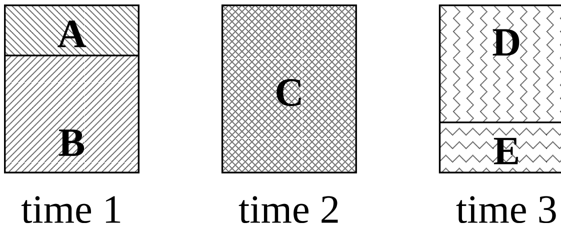


Figure 2: Field Division Example

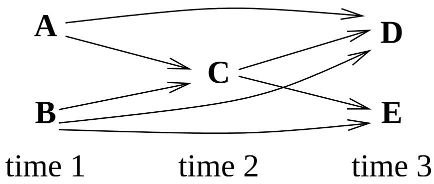


Figure 3: Overlap relation

One possible relationship between the entities in this spatial example, is to relate x to y if x precedes y and they overlap. This relation is shown diagrammatically in figure 3. Note that this relation is not transitive: A overlaps C and C overlaps E, but A does not overlap E. There are however other relations between the entities. One possibility is to relate x to y if x precedes y and x is present in the history of y. We can justify the transitivity of the relation in this example. A is involved in the history of E in the sense that the existence of E depends upon the existence of C from which it was created and C was itself created from two entities one of which was A. This relation is shown in figure 4.

To give another example consider the evolution of a family over time as illustrated in figure 5. In this example, the family at time t1 consists of just Jim and Sue. At time t2 their son, Sam, is the third member of the family. By time t3 Sue and Jim have separated and Sue is not considered part of the family, but Liz, Sam's partner is. By the final time t4, Sam has died, but

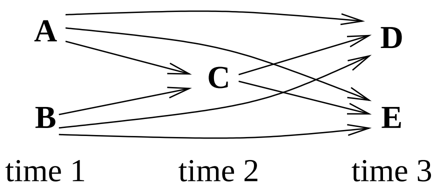


Figure 4: Historical involvement relation

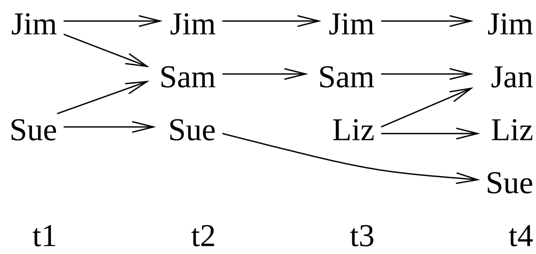


Figure 5: Family example

Sue has rejoined the family and Jan, the daughter of Sam and Liz, is also present. Figure 5 provides a convenient diagram, but some of the relationships are indicated rather than being shown explicitly. For example, Jim at time t1 is the same person as Jim at time t4, but there is no arrow between these entities in the diagram. However, the connection can be deduced from the sequence of three three separate arrows between the entities. This works in this case because the relation of either being the same person as or being an ancestor of, is transitive.

In the formal development, we use a relation called support, which covers the examples of figures 4 and 5. The idea being that an entity a supports an entity b if a exists before b and the existence of a is necessary for the existence of b. The sense in which 'necessary' is meant is not fixed by the theory, but can vary from one application of it to another. However it is interpreted, the resulting relation will be transitive. If a supports b and b supports c, then a supports c, for c cannot exist without a as a is needed for the existence of b and b is needed for c.

1.2 Overview of the Chapter

The topic of this chapter is the development of a formal model of evolving entities and, in particular, how the model can be given at various levels of detail. The ultimate application of the theory is towards spatio-temporal data in general, and geographical data in particular, but the earlier parts of the development are independent of the particular kinds of entities which change over time.

There have been many proposals for the classification of types of change which can occur in particular kinds of spatio-temporal data. Examples include changes in the content and presentation of geographic data when it is viewed at various levels of detail, changes in boundaries of land parcels in cadastral systems, and changes in relationships between relations between spatial regions when the regions are subjected to continuous change. The motivation for work of this type is to provide fundamental building blocks out of which all changes of a particular kind can be constructed. Such fundamental building blocks can be important both at the conceptual level, for the analysis of processes of change, and in a practical context, as a means of guiding the implementation of systems which record changing data. This chapter demonstrates that the concepts of amalgamation and selection can be used as the basis of a theory of granularity in spatio-temporal data. A formal model is built up, starting from just abstract sets, adding in variation over time, and adding in further features such as classifications of objects and spatial attributes.

An overview of related work in the area is given in section 2 below. The technical content of the chapter starts in section 3. That section provides the formalization of dynamic sets, which depend on the concept of a time domain. The following section, 4, contains a discussion of granularity for sets based on [SW99]. This is extended to deal with granularity for dynamic sets in section 5. The next two sections, 6 and 7, extend the basic concept of dynamic sets to cover the cases where the entities have a thematic classification, and where they have a spatial location. Some aspects in these two sections are only sketched, and more work will be required to provide a complete picture. The chapter concludes with a short summary and suggestions for further work.

2 Related Work

The subject of cartographic generalization has been widely studied, and has produced schemes which classify types of change in the representation of data on maps at different scales. The focus of this chapter is however, not on the changes to which representations may be subjected, but on changes to the data itself. This is usually referred to as **semantic generalization**.

Timpf in her thesis [Tim98] identified three kinds of hierarchies: aggregation, filtering and generalization. Each of these types of hierarchy corresponds to a means by which less detailed data may be formed. In aggregation a set of entities is formed into subsets of entities which are indistinguishable at the coarser level of detail. In filtering some entities are selected from a set, and others are omitted. Generalization in Timpf's sense is effectively a kind of aggregation of trees, in which nodes are grouped together in a controlled way. Concepts which correspond exactly to Timpf's aggregation and filtering on sets are discussed by Stell and Worboys in [SW99]. They describe notions of amalgamation and selection not only for sets, but also for graphs. Further work on graphs at different levels of detail and their application to discrete spaces, can be found in Stell's papers [Ste99, Ste00]. Another approach to classifying kinds of change that can occur when level of detail is varied is due to Puppo and Dettori[PD95]. They deal with simplification mappings between cell complexes, and identify various types of simplification.

Changes in relationships between spatial entities can occur over time as the entities move or evolve. The possible transitions between relationships which can be observed when continuous motion is involved have been classified for several systems of spatial relations, including the RCC5 and RCC8 [CB$^+$97]. These classifications lead to the continuity networks for these systems, and subgraphs of these networks correspond to the conceptual neighbourhoods originally introduced by Freksa. This work has been extended to deal with regions which are not crisp but have indeterminate boundaries [CF97, CG96]. One analysis of temporal changes in a cadastral application is due to Spéry, Claramunt and Libourel [SCL99]. Five elementary changes in the boundaries of land parcels are identified.

Hornsby and Egenhofer have proposed a change description language [HE97] capable of modelling changes over time. In later work [HE99] they have addressed views at various levels of detail of objects which are subject to change. This uses a lattice of levels of detail in a similar way to Stell and Worboys stratified map spaces [SW98]. Most recently Hornsby and Egenhofer [HE02] have considered levels of granularity in the description of moving objects.

Medak [Med99] has proposed a formal approach to change over time in spatio-temporal databases. This uses four basic operations affecting object identity: create, destroy, suspend and resume. Algebraic rules for these operations are presented, and an implementation in the functional programming language Haskell is given.

In the present book, the most closely related material is found in Smith and Bittner's *Theory of Granular Partitions.* There are clear connections between granular partitions and the classified dynamic sets in section 6 below. However, the classifications used in the present chapter need not have the form of trees which Smith and Bittner require, and their chapter does not address the partitioning of changing entities, although the theory they present is capable of being extended to treat this case. It would certainly seem worthwhile to investigate whether the concepts for a theory of granularity of change presented in this chapter could be combined with the notion of granular partition to provide a unified account of granularity and change.

3 Formalization of Evolving Entities

To provide a theory of granularity of change, we have to fix first on a model of change at a fixed level of detail, and before this we need a model of time, over which entities may evolve. A simple model of time is given in 3.1 below. This is followed by the introduction of the notion of dynamic set which is a key concept in this chapter. It turns out that the most appropriate way of defining a dynamic set is to introduce, in 3.3, the notion of a presentation, which corresponds directly to convenient diagrams of entities evolving over time. In the formal account, these diagrams, or their formalizations, are not themselves dynamic sets, only a way of describing them, and there are two equivalent ways of defining exactly what a dynamic set is. These two ways are given in 3.4, and section 3.5 shows how fundamental changes to an evolving entity are related to the formal model.

3.1 Time Domains

A **time domain** $(T, \leq)$ consists of a finite set T which carries a partial order, $\leq$. If $t, t' \in T$ where $t < t'$ and there is no t'' such that $t < t'' < t'$, then we say that

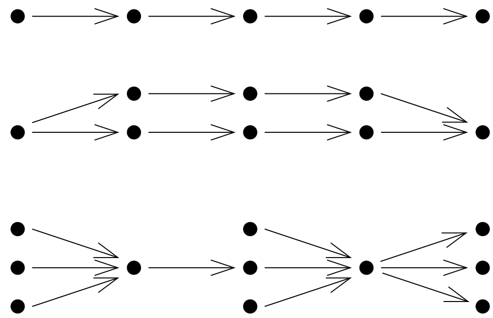


Figure 6: Examples of time domains

t and t' are **adjacent** time points. Some examples of time domains are shown in figure 6. Although the examples discussed later in this chapter are of linear time, the formal framework is not restricted to the linear case. Non-linear time can be used to model alternative accounts of the past or alternative projections of the future.

3.2 Dynamic Sets

Given a time domain T, a **dynamic set over** T models the idea of a set of objects which evolve over time. The question of when two entities at different times should be regarded as being the same object is a long-standing and difficult philosophical issue [Gal98]. In practical instances we can assume that such issues are settled on the basis of particular application needs and and are not part of a general formal framework such as this. As discussed in the introduction, we do make use of a relationship between entities at different times, which we call the supports relation. The idea is that if entities a and b exist at times t_1 and t_2 respectively and where $t_1 < t_2$, then a supports b if the existence of b depends on the existence on a in some sense. The particular interpretation of the supports relation will depend on the application, it might for example be some kind of causal dependency. There are other possibilities, the coarsest being that any entity supports all later entities. This can be interpreted as a dependency of existence on the view that the state of the world at any time is a consequence of everything that has happened previously.

3.3 Presentation

Consider the picture of figure 7. The picture shows five successive times t1 to t5, above each of which is a set of entities, to be thought of as the entities existing at that time. The entire set of entities, or more correctly the disjoint union of each of the sets above a time point, bears a relation. Since the relation follows time in moving from left to right in the picture, related pairs are shown joined by a line without an arrow head. Such a picture is a way of presenting

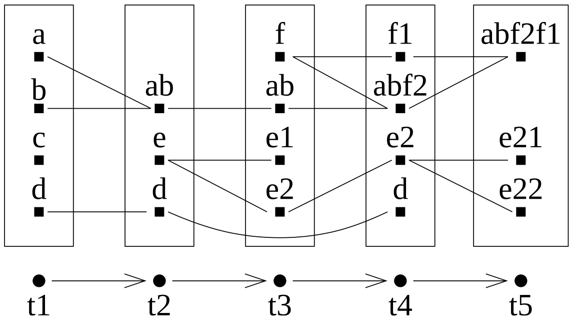


Figure 7: Presentation of a dynamic set

the supports relation, and to explain how we need to recall the concept of the covering relation of a poset [Grä98, p12].

Given a poset $(A, \leq)$, we define the relation $\prec$ by $a \prec b$ if $a < b$ and there is no x for which $a < x < b$. When A is finite, the relation $\prec$ completely determines the relation $\leq$, the passage from $\prec$ to $\leq$ being given by taking the reflexive transitive closure. The covering relation of a poset will be acyclic, in the sense that in every chain $a_1 \prec a_2 \prec \cdots \prec a_n$ all the a_i are distinct (observe that this includes the property of being irreflexive, i.e. $a \prec a$ never happens). The covering relation is also intransitive, i.e. if $a_1 \prec a_2 \prec a_3$ then $a_1 \not\prec a_3$. Calling a finite relation with these properties a **cover**, then there is a one-to-one correspondence between covers and finite posets, and it is convenient to specify a finite poset by giving its cover.

Thus covers provide a means of presenting posets, and the diagram of figure 7 consists of two covers, one for the supports relation, and one for the time domain, together with a mapping between them. To describe the mapping we need to consider morphisms between covers. The most general kind of morphism which we require between covers A and B is an order preserving map of relations between A and B^*, the reflexive, transitive closure of B. This is a special case of the morphisms between graphs used in [SW99], and that reference explains how such morphisms are composed. When dealing with collections of morphisms between structures, the notion of a category is a useful tool, and for basic category theory, the book of Barr and Wells [BW95] will be found helpful. The category where the objects are covers and the morphisms are the ones just mentioned, will be denoted **Cover***. We also need to make use of a restricted kind of morphism of covers $f : A \to B$, where if $a_1 \prec a_2$ in A then $fa_1 \neq fa_2$ in B, these morphisms will be called **irreflexive**.

Now we can define formally a **presentation of a dynamic set** to consist of two covers A and T together with an irreflexive morphism $\rho : A \to T$. In figure 7 the effect of the morphism is shown by the vertical alignment in the diagram.

3.4 Formalizations of Dynamic Set

There are two equivalent ways of formalizing the notion of dynamic set, that is, of specifying what exactly it is that the presentations just discussed present. One, which we introduce first, takes a global set of entities equipped with the supports relation and provides a function assigning to each its time. The second provides for each time a set of entities and instead of one relation, a collection of relations, indexed by pairs of times in temporal sequence.

Dynamic sets have some connections with Kripke models [Mit96], but these generally have functions, rather than arbitrary relations between the sets at successive possible worlds (times). The idea of Kripke models where the poset of possible worlds is considered at multiple levels of detail (as in section 5) does not appear to have received much attention in the literature.

3.4.1 Global Formalization

A **dynamic set over** a time domain T consists of a poset $(A, \leq)$ and an order-preserving function $\rho : A \to T$, such that if $\rho a_1 = \rho a_2$ and $a_1 \leq a_2$ then $a_1 = a_2$. The idea behind this restriction is that at a fixed time, an entity supports only itself. Given a presentation $\rho : (A, \prec) \to (T, \prec)$ we obtain a dynamic set $\rho : (A, \leq) \to (T, \leq)$ by taking $(A, \leq)$ and $(T, \leq)$ to be the reflexive-transitive closures of $(A, \prec)$ and $(T, \prec)$ respectively. The function ρ given in the presentation is the same (as a function on the underlying sets) as that required for the dynamic set itself.

3.4.2 Local Formalization

A **dynamic set**, X, **over** T consists of for each $t \in T$, a set $X(t)$, and whenever $t \leq t'$, a binary relation $X(t, t')$ between the sets $X(t)$ and $X(t')$. These relations must satisfy the condition that if we have times t, t', t'' with $t \leq t' \leq t''$ and $x \in X(t)$, $x' \in X(t')$, and $x'' \in X(t'')$ with x related to x' by $X(t, t')$ and x' related to x'' by $X(t, t')$, then x must be related to x'' by $X(t, t'')$. We also require that $X(t, t)$ is the identity relation on $X(t)$. These conditions can conveniently be summarized by regarding T as a category, and specifying that X be a lax functor from this category to the category of sets and relations. A useful reference for the concept of lax functor is [KS74].

Given a dynamic set $\rho : A \to T$ in the previous formulation, we can obtain the equivalent local structure by taking $X(t)$ to be $\{a \in A \mid \rho a = t\}$, and defining $X(t, t')$ by restricting the relation on A to the subset of A given by $X(t) \cup X(t')$.

3.5 Fundamental Changes

If t and t' are two immediately succeeding times, there are four fundamental changes which can occur between the elements of $X(t)$ and the objects of $X(t')$.

death or suspension An object may be present at t but have no corresponding object at t'. For instance, the element $c \in X(t1)$ in the dynamic set presented in figure 7, and also $d \in X(t2)$.

birth or resumption An object may be present at t' but have no corresponding object at the previous time. For example $d \in X(t4)$ in figure 7 and also $e \in X(t2)$.

merge Two or more objects at the earlier stage combine to form an object at the subsequent stage. For example a and b in $X(t1)$ combine to form the object denoted by ab in figure 7.

split One object at the earlier stage splits into two or more objects at the later stage. This can be seen in the splitting of $e \in X(t2)$ into $e1$ and $e2$ in $X(t3)$.

The absence of each one of these kinds of change corresponds to one of four basic properties which a binary relation may have. If no objects die or suspend, the relation is total; if no objects are born or resume, it is surjective; if no objects merge, it is injective; if no objects split, it is functional.

It is noteworthy that there can be times t, t', t'' where $t < t' < t''$ with $x \in X(t)$ related to some $y \in X(t'')$ but not related to any element of $X(t')$. An instance of this appears with d in figure 7. This corresponds to having an object which exists at times t and t'' but is in some sense non-existent in between these two times. Well known examples include countries or states which have had multiple episodes of existence through history, for example Austria. From a formal viewpoint, this behaviour is allowed by permitting X, considered as a functor, to be lax.

4 Granularity for Static Sets

4.1 Amalgamation and selection

In this section we examine what it means for one set to be a less detailed version of another set. Although abstract sets are the simplest of structures, they provide the basis on which more elaborate models are constructed.

The discussion does not require that the elements of the sets which we consider in this section have any specific properties. However, it is useful for the eventual applications of this work to bear in mind that the elements might be objects which could be stored in a spatial database or a GIS. Examples of such objects could be a specific building, a road, a lake, or a town etc. The data held in a spatial database can be envisaged, at a conceptual level, as a set of objects each of which may be equipped with both spatial and non spatial attributes.

Given a set, X, of objects, a less detailed representation of X will be a set Y formed from X by the combination of two processes.

- Some of the objects of X are selected to contribute to Y, and others are rejected.

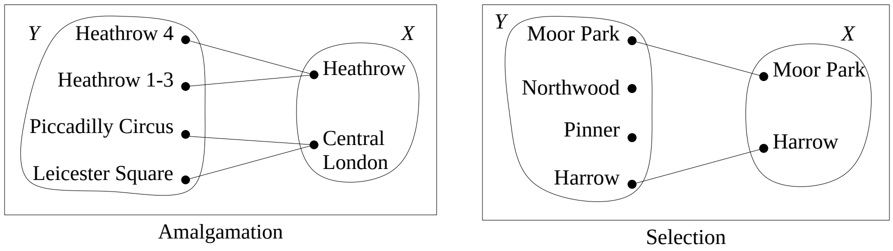


Figure 8: Loss of detail between sets

- The selected objects may subjected to an equivalence relation so that distinctions between some objects are forgotten.

The two operations are illustrated in figure 8. In the amalgamation example, the set Y consists of four stations on the London Underground. For some application it may be inappropriate to distinguish between the two individual stations at Heathrow Airport. Similarly, the stations Piccadilly Circus and Leicester Square are physically close together, and at a lower level of detail, the distinction between them may not be important. By avoiding the distinctions between these pairs of stations, we arrive at the set X as a less detailed representation of the data in Y.

The example of selection is also derived from actual data about the London Underground. Here again Y is a set representing four individual stations which is represented at a lower level of detail by a set X containing only two elements. However, in this case the operation performed on Y to produce X is quite different. The stations present in X are selected from those in Y because of their relative importance. Northwood and Pinner are minor stations, and many trains which do stop at Moor Park and Harrow do not stop at the two smaller stations.

When X and Y are sets it is straightforward to formalize the notions of amalgamation and selection. If the relationship of X to Y is one of selection, then there is an injective (or one-to-one) function from X to Y. If the relationship is one of amalgamation, then there is a surjective (or onto) function from Y to X.

4.2 Combining Amalgamation and Selection for Sets

The above examples deal with two ways in which X may be a less detailed representation of Y. In more complicated examples the relationship need not be solely one of amalgamation or selection. In general, a loss of detail relationship between X and Y will involve both selection and amalgamation. This entails a set Z which is obtained from Y by selection, and which is amalgamated to produce X. A simple example appears in figure 9. A pair consisting of a selection followed by an amalgamation will be called a *simplification* from Y to

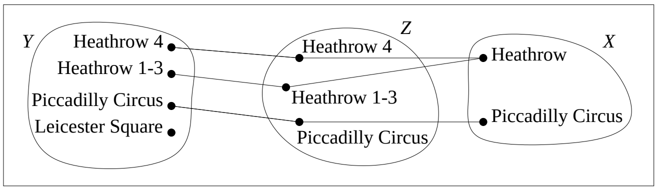


Figure 9: Combined Amalgamation and Selection

X. Formally, a simplification from a set Y to a set X consists of three things: a set Z, an injective function from Z to Y (the selection part) and a surjective function from Z to X (the amalgamation part). Alternatively we can describe a simplification from Y to X as a partial surjective function from Y to X.

It might appear that by defining a simplification to consist of a selection followed by an amalgamation, we are being unnecessarily restrictive. It is natural to ask whether this definition of simplification excludes an amalgamation followed by a selection, or a sequence of the form $[s_1, a_1, s_2, a_2, \cdots, s_n, a_n]$ where each a_i is an amalgamation, and each s_i is a selection. In fact, provided we are dealing with simplifications of graphs, or of sets, every sequence of the above form can be expressed as a single selection followed by a single amalgamation. The justification for this lies in the fact that simplifications can be composed.

It is worth noting that a single selection on its own is still a simplification. This is because it can be expressed as a selection followed by the trivial amalgamation in which no distinct entities are amalgamated. Similarly, a single amalgamation on its own is a simplification, since it is equal to the trivial selection, which selects everything, followed by the amalgamation.

4.3 Simplification as Amalgamation and Selection

A selection from a set X is a set Z and an injective function from Z to X. An amalgamation of a set X is a surjective function from X to a set Z. In general loss of detail can involve both selection and amalgamation, so a **simplification** from X to Y is defined to consist of a set Z, and functions $f : Z \to X$ and $g : Z \to Y$ where f is a selection and g is an amalgamation. An alternative description of a simplification from X to Y is a partial function from X to Y which is surjective. This view is often technically simpler, and makes it clear that simplifications can be composed, but it is not as conceptually useful in that twin concepts of selection and amalgamation are not made explicit.

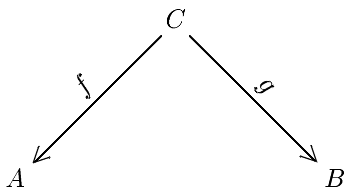
5 Granularity for Dynamic Sets

The previous section considered simplifications of sets in terms of amalgamation and selection. We now extend this from simple static sets to dynamic ones.

5.1 Simplification of Covers

A cover was defined in section 3.3 as a relation on a finite set which is intransitive and acyclic. If $(A, \prec)$ and $(B, \prec)$ are covers, then a function $f : B \to A$ is a **selection** if it is injective and if $b_1 \prec b_2$ then $fb_1 \prec^+ fb_2$ where $\prec^+$ is the transitive closure of $\prec$. In view of the injectivity condition, we could equivalently put the reflexive-transitive closure in the definition of selection. A function $f : B \to A$ is an **amalgamation** if it is surjective and $b_1 \prec b_2$ then $fb_1 \prec fb_2$.

A **selection** from a cover $(A, \prec)$ to a cover $(B, \prec)$ is defined to consist of a cover $(C, \prec)$ together with a selection f and an amalgamation g as in the following diagram in the category **Cover***.



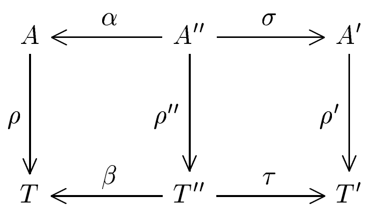
The definition of amalgamation does not allow c_1 to become identified with c_2 if $c_1 \prec c_2$. This is not restrictive when working with simplifications as if an identification is required the selection can be constructed so that the entities to be identified are unrelated by the $\prec$ relation in C.

Describing simplifications in this form is conceptually useful as it emphasizes the separate components of selection and amalgamation. However, it is often useful to use an equivalent description of a simplification from $(A, \prec)$ to $(B, \prec)$ as a partial function from A to B which is surjective and order preserving in the reflexive-transitive closure of the relations, but not necessarily order preserving in the relations themselves.

5.2 Granularity for Dynamic Sets

We now consider what should be meant by a simplification of a dynamic set. It is easiest to define this concept in terms of presentations rather than the dynamic sets themselves.

A simplification from (a presentation of) a dynamic set $\rho : A \to T$ to $\rho' : A' \to T'$ consists of a dynamic set $\rho'' : A'' \to T''$, with simplifications of A to A' and T to T' such that the following diagram in **Cover*** commutes.



Some of the situations that are catered for by this definition are illustrated in figures 10 to 14. In figure 10 the two time domains are identical. The effect

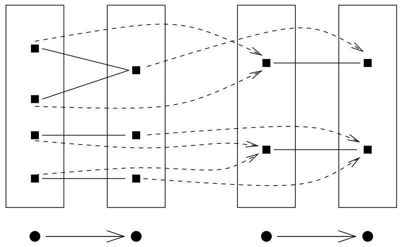


Figure 10: Amalgamation of entities

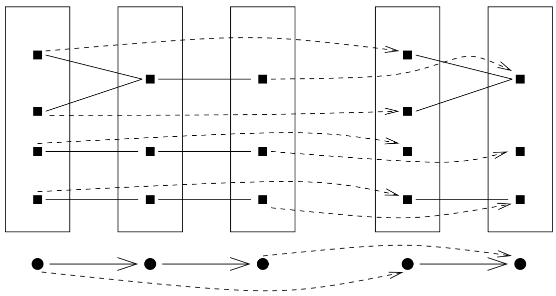


Figure 11: Simplification involving omission of a time point

of the simplification on the entities is indicated by the dashed lines. This figure shows that entities of the same time may be amalgamated by a simplification, and that the images of two entities unrelated in the support relation may become related after the simplification.

In figure 11 the simplification of the dynamic sets includes a simplification of the time domain. In this case only two of the times in the original domain are selected and the middle one is omitted. In this situation all the entities existing at the omitted time must themselves be omitted too, but the formal definition allows flexibility as to whether relations of support via these omitted entities are maintained in the simplified dynamic set or omitted.

Figure 12 illustrates the omission of entities and of support relations which can occur even when the simplification of the time domains is the identity. Observe in particular that where we have three entities where a supports b which supports c, we may omit b and still keep a supporting c. In terms of the presentations we are working with, this means that the simplified dynamic set presentation will show a link from a to c even though the original presentation showed no such link. However, in the dynamic sets presented by these presentations a supports c in both cases.

The formal definition of simplification permits a supporting b to be simplified to a not supporting b, but only allows a not supporting b to be simplified to a supporting b in very special circumstances, which are illustrated in figure 13.

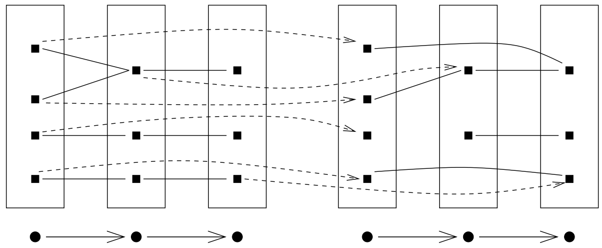


Figure 12: Omission of entities and support connections

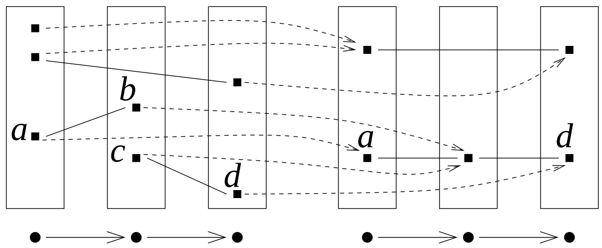


Figure 13: Addition of new support through simplification

This asymmetry in which connections of support my be lost, but not in general added fits the notion of simplification. That is, simplification involves the loss of information (in this case that something supports something else) but not the arbitrary addition of new information.

As shown in figure 13 new connections of support may be created through the requirement that the supports relation is transitive. The figure presents a situation where neither a nor d is itself involved in an amalgamation with another entity, and a and d are initially unrelated in the supports relation. However, since a supports b and c supports d, and b becomes identified with c in the simplification, a does support d in the dynamic set resulting from the simplification.

Figure 14 shows what can happen when the simplification of the time domain amalgamates two distinct times into one. In this case the definition of simplification of dynamic sets forces some identifications between the entities existing at the times which are identified. If entities a and b exist at two times which become identified, then a and b must them selves be identified if we can find a sequence of support connections (ignoring the direction of the supports relation) which link a to b. The figure also demonstrates that entities taken from each of the sets above the two identified times may or may not become identified themselves when they are not connected by a sequence of support connections.

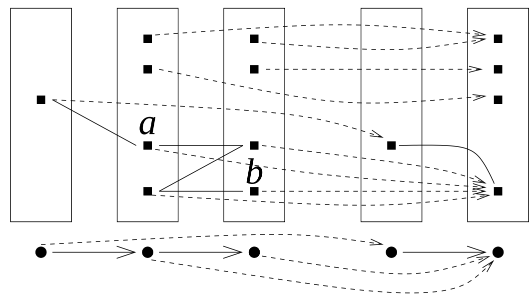


Figure 14: Simplification with amalgamation of two times

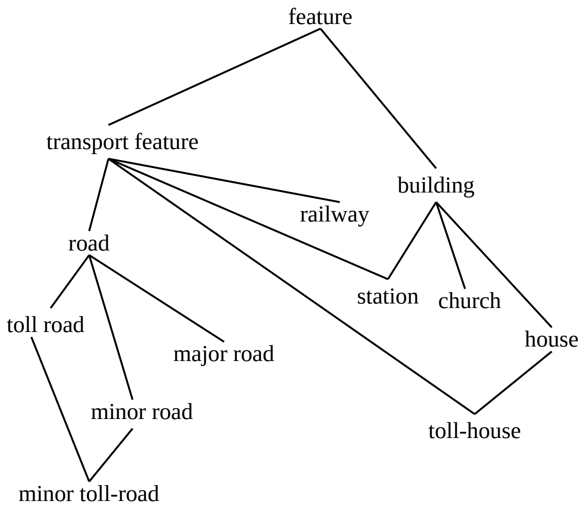


Figure 15: A classification hierarchy

6 Classified Dynamic Sets

6.1 Classifying Sets

So far the objects which vary over a time domain have just been modelled as elements of sets. To introduce more sophisticated models we can add a classification to each object. This allows us to identify some objects as roads, others as railways and yet others as houses. The classification attached to an object can vary over time, for instance a minor road may become a major road, or a railway may become disused and turned into a cycle track. However there are generally limits to changes, for instance a house cannot become a railway, nor a tree a river. The classifications can be arranged into a hierarchy, or possibly several hierarchies, as indicated in figure 15. A **classification** $(\Phi, \sqsubseteq, \succcurlyeq)$ consists of a set Φ partially ordered by $\sqsubseteq$ and equipped with a reflexive and transitive relation $\succcurlyeq$. It is required that any set of elements of Φ lying in the same connected component with respect to $\sqsubseteq$ has a least upper bound. It is also required that if $\varphi_1 \succcurlyeq \varphi_2$ then there is some ψ such that $\psi \sqsubseteq \varphi_1$ and $\psi \sqsubseteq \varphi_2$. The idea is that if $\varphi_1 \sqsubseteq \varphi_2$ then φ_1 is a more general concept than φ_2, for example animal and dog respectively. The relation $\succcurlyeq$ models the idea that instances of some concept can evolve to instances of another concept. For example, a puppy can over time become a dog but not an elephant, even though all are instances of animal. A **classified dynamic set** is defined to be a dynamic set X over a time domain T together with for each $X(t)$ a function $\eta_t : X(t) \to \Phi$. It is required that if $t \leq t'$ in T and $x \in X$ is related by $X(t, t')$ to $x' \in X(t')$ then $\eta_t x \succcurlyeq \eta_{t'} x'$.

6.2 Temporal Simplification of Classified Dynamic Sets

If we have a dynamic set X over a time domain T, and X is classified by Φ, where $\eta_t : X(t) \to \Phi$, any simplification of time domains $\sigma : T \to S$ induces a simplification of classified dynamic sets. That is, we can construct a dynamic set Y over S which is also classified by Φ.

To define the dynamic set Y we need to specify

1. For each element s of the time domain S, a set $Y(s)$.
2. For each pair of elements s and s' of S where $s \leq s'$, a relation $Y(s, s')$ between $Y(s)$ and $Y(s')$.

For each $s \in S$, $\sigma^{-1}s$ is a subset of T. Define $W(s)$ to be the disjoint union of the family of sets $(X(t))_{t \in \sigma^{-1}s}$. The set $Y(s)$ is formed from $W(s)$ as the equivalence classes of the equivalence relation which makes $x \in X(t)$ equivalent to $x' \in X(t')$ whenever x and x' are related by the equivalence relation $\sim$, generated by the requirement that $x \sim x'$ if and only if x is related to x' in $X(t, t')$. The relations $Y(s, s')$ are defined by requiring that $y \in Y(s)$ is related to $y' \in Y(s')$ iff there are $x \in y$ and $x' \in y'$ where $x \sim x'$.

All we need to do now is specify how each of these sets is classified. An element of $Y(s)$ is an equivalence class of elements of sets of the form $x \in X(t)$ where $t \in \sigma^{-1}s$. The set $A(s) = \{\eta_t x \in \Phi \mid x \in y, \text{ and } x \in X(t)\}$ will consist of elements of Φ all lying in the same connected component. Thus the least upper bound of $A(s)$ will exist, and we classify $Y(s)$ by $\theta_s : Y(s) \to \Phi$ where $\theta_s(y)$ is the least upper bound of $A(s)$.

To illustrate this process, figure 16 shows an example of a dynamic set classified by the hierarchy in figure 15. The symbols have the following meaning shown in figure 17. In the dynamic set at time t1 we have a toll-house, a minor toll-road and a track. By time t2 the toll-house has been divided into two ordinary houses, and the toll-road and track are unified into a single minor road. By time t3, one of the two houses has gone and the other has been converted into a church. The minor road is now merely a track.

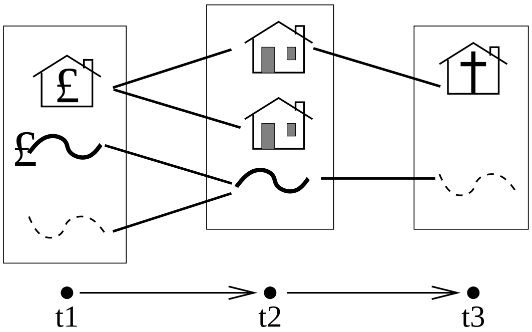


Figure 16: A classified dynamic set

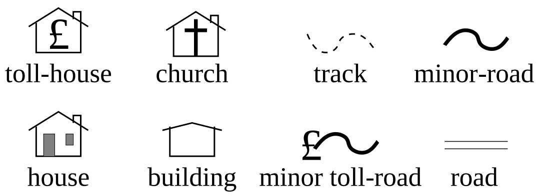


Figure 17: Key to the symbols

Figure 18: Two induced simplifications

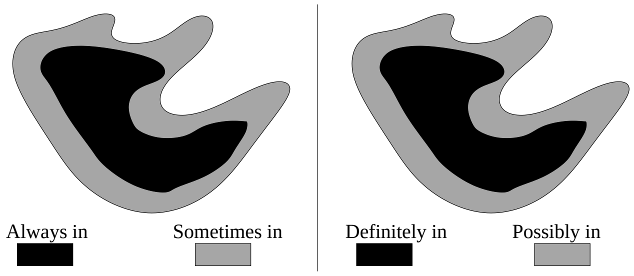


Figure 19: Interpretations of egg-yolk regions

Two different induced simplifications of this situation are shown in figure 18. In the left hand example, times t1 and t2 are amalgamated. At this level of detail, all we can say is that there is a road of some kind and a house. It is not possible to distinguish between the three kinds of road which are apparent at the more detailed level. In the right hand example, one of the two houses is amalgamated with the church, and the most detailed classification possible is that there is a building, but the granularity prevents a more detailed classification.

7 Spatially Referenced Data

In a dynamic set X over T, the elements of each $X(t)$ can be given an associated spatial region as well as a classification in a hierarchy. This amounts to adding a temporal dimension to the approach proposed by Stell and Worboys [SW98] which was based on the concept of a stratified map space. Thus we use a formal framework where R is some set of regions. These regions might be regular closed sets in the plane, but there are other possibilities, such as the discrete regions discussed in [Ste00].

In the context of reduction in temporal level of detail egg-yolk regions as discussed in [CG96] can be used. An 'egg-yolk' region consists of a pair of regions (x, y) where x is a subregion of y. The original formulation of egg-yolk regions required that x be a proper part of y, but for the application here, it is more appropriate to allow x to be equal to y. The set of egg-yolk regions constructed from a set of regions, R, will be denoted $R^{\circledcirc}$.

Egg-yolk regions are often used to represent regions with indeterminate boundaries. With this interpretation, (x, y) can be used to stand for a region which we know contains x and which is contained in y, but the precise location of which is uncertain. Egg-yolk regions can also be used to represent regions which vary over time. In this interpretation, (x, y) can stand for a region which always contains x and where anywhere in the difference $y - x$ is sometimes but not always in the region. These two interpretations of an egg-yolk region are illustrated in figure 19. A simple example of how temporally indeterminate regions arise from change in level of detail appears in figure 20. Here we have a

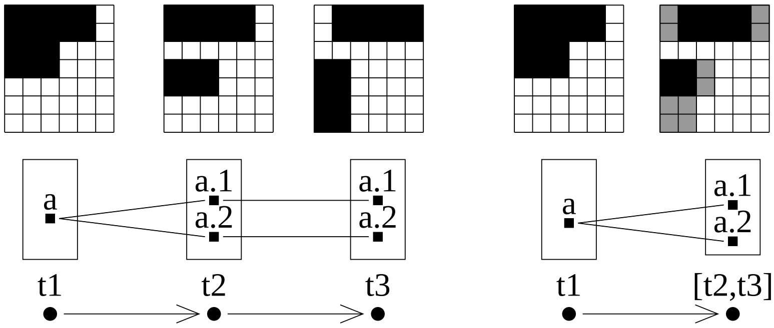


Figure 20: Indeterminacy from spatio-temporal granularity

dynamic set X over a three element time domain $T = \{t1, t2, t3\}$. The set $X(t1)$ consists of a single element, a, which has an associated region shown above it in the diagram. By time $t2$, the entity a has divided into a.1 and a.2 each of which has an associated region. By time $t3$ the regions associated to the two entities have moved to a different location. On the right of figure 20 is shown the result of a simplification of the time domain which amalgamates $t2$ and $t3$. Here the region associated to a.1 is an egg-yolk region. The yolk is the intersection of the regions associated to a.1 at the two times, and the egg is the union of the two regions. The egg-yolk region associated to a.2 is constructed similarly.

To describe the general situation, suppose we have a dynamic set X over a time domain T which is simplified to S. A given $s \in S$ will result from the amalgamation of a set of elements of T, say $Q(s)$. The construction of the simplified dynamic set Y over S from X has been described earlier. The additional structure here is that each $X(t)$ is equipped with a function $\rho_t : X(t) \to R^{\circledcirc}$. We are deliberately using $R^{\circledcirc}$ rather than R here in order to cope with repeated simplifications. Also the decision to allow egg-yolk regions (x, y) where $x = y$ means that any region in R can be regarded as element of $R^{\circledcirc}$, thus the example given above is included in our framework. It remains to show how each $Y(s)$ is equipped with a function $\sigma_s : Y(s) \to R^{\circledcirc}$. Each element $y \in Y(s)$ will result from the amalgamation of a set of elements each of which belongs to some $X(t)$ where $t \in Q(s)$. For each $t \in Q(s)$ let $A(t, s)$ denote the subset of $X(t)$ which is amalgamated to form y. Then the function σ_s is defined by

$$\sigma_s(y) = \left(\bigcap_{t \in Q(s)} \bigcup_{x \in A(t,s)} \downarrow \rho_t(x), \bigcup_{t \in Q(s)} \bigcup_{x \in A(t,s)} \uparrow \rho_t(x) \right)$$

where $\downarrow \rho_t(x)$ and $\uparrow \rho_t(x)$ denote respectively the yolk and the egg of $\rho_t(x)$.

8 Conclusions and Further Work

This chapter has presented a formal model for data varying over time based on a set of entities at each time and relations between these sets. The simple concept of a relation between sets is able to deal with most of the important changes to which entities can be subjected. These include birth, death, suspension of existence, resumption of existence, merging and splitting. By equipping these sets of entities with functions to classification hierarchies and to sets of egg-yolk regions it is possible to deal with entities having spatial attributes and non-spatial classifications.

There are several areas for further work. One topic which could be investigated in this framework would be topological relations between regions varying over time and their description at varying levels of detail. For example, if regions are disjoint at time t1 and partially overlapping at t2 we could ask for a description of their relation when the times t1 and t2 are amalgamated.

Another topic would be the incorporation of spatial indeterminacy into the framework. That is, the regions could not only vary over time, but also be of uncertain location. This would appear to need a more elaborate kind of region than the egg-yolks used in section 7 above.

Acknowledgements

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