“Though this be madness, yet there is method in it [*]*”*: The Importance Of Mathematical Concepts Beneath Contemporary Visualization.

David Banks[†]
Univ. of Tennessee
www.jics.utk.edu

Hamish Carr[‡]
Univ. College Dublin
www.cs.ucd.ie

Roger Crawfis[§]
Ohio State Univ.
www.cse.ohio-state.edu

Tamara Munzner[¶]
Univ. of British Columbia
www.cs.ubc.ca

Valerio Pascucci[‖]
Lawrence Livermore Nat. Lab.
www.llnl.gov

Rita Borgo[**], **David Duke**[**] (moderators)
School of Computing, University of Leeds
www.comp.leeds.ac.uk

Index Terms: G.0 [Mathematics of Computing]: General—Project and People ManagementLife Cycle; I.3 [Computing Methodologies]: Computer Graphics—;

1 INTRODUCTION

When asked what he thought of Western civilization, Mohandas Gandhi is reported to have replied that he thought it would be a good idea. How would a mathematician respond if asked about the mathematical basis of visualization ... and does the answer matter?

Visualization occupies an important role in the pursuit of knowledge as the understanding of real world phenomena. From an external viewpoint, images assist mental processes like perception of spatial relationships, comprehension of complex time-varying behaviours, and detection of patterns. However, underlying the image are mathematical models of the world and its representation. Some of these models are obvious and relatively well-known; these include the calculi of scalar/vector/tensor spaces, the study of manifolds and homotopies. Others are (or at least their applicability to visualization) less well known: multivariate representation, infinite sequences, complex functions, and group theory which have all contributed novel insight into the nature of data and representation, or methods for transforming data into truth-preserving images. The key issues facing the visualization discipline - from the “madness” of ever increasing dataset scale, to new challenges of visualizing multi-scale biological systems or huge volumes of streaming real-time data - call for an improved understanding and appreciation of its underlying mathematics. Further, at a time when questions about the very core of the discipline, e.g. the relationship between information and scientific visualization, are being raised almost annually within the conference, it seems particular germane to stand back and review what is ultimately what could be the most abstract and neutral picture of visualization itself - the mathematics world which ties together the notion of data and transformation.

This panel brings together researchers who have been pioneering the fruitful relationship between mathematics and visualization by integrating visualization techniques in mathematics [4, 7, 5], as well as mathematical theory and methods that are used for visualization [2, 1, 3]. The panelists will present their views and experiences on fundamental mathematical research in visualization and vice versa, and will address limits, challenges and potentials of this alliance. The list of assessments will include, but not be limited to, topics like the exploitation of mathematical models to improve quality and comprehension of visualization, the quest between realistic results versus aesthetic representations, the possibility to ease the extraction of ontologies and methodologies shared by apparently different classes of visualization algorithms, the bi-directional drive that mathematics for visualization and visualization of mathematics can provide to the challenge and understanding of real-world problems.

The panel will involve lively discussion and debate on the kinds of mathematics applicable to visualization and the means by which maths is brought into the discipline. After summarizing the ‘case for the defense’, members of the panel will be cross-examined by the audience on whether their own work is the madness of genius, or the product of mathematical methods. The deeper question, however, is whether a mathematical civilization beats at the heart of the visualization community.

2 POSITION STATEMENTS

David Banks
“Mathematics In Visualization”

Many 3D datasets are organized as a grid of scalar values. The grid forms a discrete sampling of the spatial domain R3, while the scalar quantities are evaluations of a mapping $f : R3 \rightarrow R$. One way to visualize the mapping is to display its level sets. A level set $L(f,c)$ is defined as the locus of points p where $f(p) - c = 0$. The Marching Cubes [6] algorithm is a widely used technique to produce a polygonal approximation of $L(f,c)$ using a pre-determined set of geometric substitutions applied to each grid cell. Each substitution depends on the “coloring” of the cell: the (abstract) color of a vertex p is either white or black, according as the value $f(p)$ is either positive or negative. The colorings can be enumerated using techniques from group theory. These techniques can be generalized to enumerate configurations found in several variants of Marching Cubes. This analysis of Marching Cubes provides an example of a mathematical tool that helps unify a class of visualization algorithms by expressing them in a common way.

[†] dbanks@cs.utk.edu

[‡] hamish.carr@ucd.ie

[§] crawfis@cse.ohio-state.edu

[¶] tmm@cs.ubc.ca

[‖] pascucci@llnl.gov

[**] e-mail:{rborgo|djd}@comp.leeds.ac.uk

Hamish Carr
"Visualization, Algorithms & Programming, And Mathematics"

Mathematics is needed to drive visualization, but visualization is also needed to drive mathematics. The basic problem of visualization (of all forms) is to take large amounts (up to 10^{15} data points or more) of data and map them to a (relatively) small number (10^8) of retinal cells and low-bandwidth (60 KB/s) transmission up the optic nerve. Visualization therefore relies on abstracting the data and converting the resulting abstraction into a visually effective representation of important features, preferably one that maps simply to the abstractions imposed by the human visual system. Since abstraction is at the core of how we understand large amounts of data visually, it is inevitable that mathematics, the systematic study of abstraction, should play a large part in the process. Thus, it is predictable that many, if not most, of the useful abstractions, have already been studied in mathematics. Which abstractions should be used depends largely on what information is most useful. For spatial information and quantitative measurements, geometry is generally most useful, but also analysis and statistics. For shape description, object recognition and the like, topology is preferred. And for functional relationships, combinatoric descriptions are generally the most useful. In all this, however, we should never lose sight of the fact that mathematics is not only about abstraction - it is principally about useful abstractions. Even though apparently useless abstractions are developed, they often turn out to be useful later, leading to a Darwinian selection of abstractions in favour of those that give the most useful models of the physical world. Identifying the most useful abstractions, however, is itself a non-trivial task. This depends on which abstractions are applicable in the first place, what their limiting assumptions are, what processing assumptions they make, how computable they are in practice, and how well they fit experimental requirements and observations.

It follows from this that, not only should mathematics be used to provide abstractions for visualization, but that visualization (among other fields) should be used to select, drive and develop abstractions in mathematics. Particular challenges at present in the geometric, topological and quantitative domains include the need to loosen "simplifying" mathematical assumptions that do not correlate well with experimental procedures, and developing mathematical representations of imprecise or uncertain calculations.

Roger Crawfis
"User Studies, Mathematical Models And Algorithms"

The past two decades of research into scientific visualization has primarily focused on improved rendering speed, data management or novel representations. Much of these improvements have come from a streamlining of the underlying mathematic models developed early on in the visualization research. For instance, various optical models were developed in the late-80's to model the propagation of light. Algorithmic performance requirements have simplified these, leading to round-off errors, popping, and other aliasing artefacts in the commonly available tools. While some of these errors are known, the complex interplay between the algorithms and the underlying mathematics is often complex. Several well-known publications have often mis-interpreted errors in algorithms as a fault of the underlying mathematical model, when in fact the assumptions made in the algorithm lead to a different mathematical model. As the performance of the visualization tools has reached near real-time levels, more research is needed to improve the quality and comprehension of the visualizations. This will occur on two fronts, user studies and better mathematical models and algorithms that support these.

Tamara Munzner
"Mathematics For Visualization Versus Visualization Of Mathematics"

I distinguish between mathematics for visualization, namely mathematics being used as a tool in service of visualization of non-mathematical information, and visualization of mathematics, namely visualization being used as a tool to help people understand mathematical ideas. I'll call the former "math for vis", and the latter "mathematical visualization" or "mathvis". Both are of course important and useful, but are aimed at different audiences and have different goals. For instance, hyperbolic geometry has been used as a tool for several information visualization systems designed to show hierarchical information, because it provides an elegant mathematical framework for handling an exponential number of items. While the mathematics involved is indeed described in papers aimed at the research community, these systems are aimed at non-technical audiences. The ideas behind the underlying hyperbolic transformations deliberately hidden by simply describing the visual effects as a kind of fisheye transformation. In contrast, mathvis is aimed at helping students of mathematics understand the implications of geometries where Euclid's parallel postulate does not hold. It emphasizes the unfamiliar and surprising aspects of hyperbolic geometry, such as the fact that translation has a center point, just like rotation in euclidean spaces, which are deliberately deemphasized in the hyperbolic tree viewers. A key force that drives infovis and scivis algorithm development, that real-world datasets created by simulation or sensors or logging are growing increasingly larger, is less relevant in mathvis.

Both infovis and scivis practitioners have a model where the central concern is helping people understand a particular input dataset, and thus an interest in developing scalable algorithms that can handle large datasets. In contrast, in mathvis the goal is to often to help people understand the implications of a particular mathematical concept, such as the characteristics of a space that is non-euclidean or higher dimensional or a manifold with its boundaries glued together with interesting twists. In these cases, often many different datasets are inspected to illustrate the properties of the space, and the size of these datasets is rarely a concern. Even when the goal is to understand the structure of a particular mathematical object, such as a specific knotted sphere in four-dimensional space, these datasets are often tiny by the standards of infovis or scivis. However, the mathvis challenge of providing insight into a space itself, rather than a particular dataset, is a challenging and sometimes elusive goal!

Valerio Pascucci
"Seeing Is Believing"

Seeing is believing. This is why visualization plays, and has always played, a central role in the communication and illustration of known scientific phenomena. The modern rise of massive scientific simulations and high resolution experimental devices poses a major challenge and opportunity: can visualization be a central component of the actual scientific discovery? The main challenge for the visualization community is not to get lost in the exclusive pursue of performance and aesthetic, but to maintain focus on the value that visualization can have as a high bandwidth data understanding tool. Good visualization tools can provide scientists with a faster way to evaluate the results of their experiments. Unfortunately too many scientists got used to adopt visualization tools only for illustration, after having wasted too much time chasing involuntary optical illusions and artefacts generated in data conversion, simplification, or re-sampling needed for visualization. Rising to the current challenge of aiding true scientific discovery will require a visualization renaissance based on (i) systematic use of rigorous mathematical models, (ii) disciplined reporting of error bounds and uncertainties, and (iii) honest explicit presentation of any limitation of the techniques adopted. This will provide great opportunities for developing new research but will also require a new modus operandi based on closer working relations with the user scientists.

During the panel discussion I will illustrate the challenges and opportunities of this process using, as an example, recent research centered on the development rigorous topological techniques for scientific visualization and their practical use for the analysis of terabytes of scientific data.

3 BIOGRAPHICAL SKETCHES

David Banks

David Banks is a senior scientist with the UT/ORNL Joint Institute for Computational Sciences with an interest in visualizing multidimensional data in science and medicine. He received his Ph.D. in Computer Science from UNC Chapel Hill and completed his post-doctoral training at NASA Langley Research Center. He recently served as visiting professor of radiology at Harvard Medical School's Surgical Planning Laboratory.

Hamish Carr

Hamish Carr is a Lecturer at University College Dublin since September 2004. Hamish Carrs research interests lie in computer graphics, scientific and medical visualization and computational geometry and topology. He is also interested in end-user applications that generate geometric and visualization problems. A recent (2004) Ph.D. graduate from the University of British Columbia, his graduate research applied topology to improve user exploration of scientific data.

Roger Crawfis

Roger Crawfis received a joint B.S in Computer Science and in Applied Mathematics from Purdue University before joining the Lawrence Livermore National Laboratory. He subsequently lead the visualization research at LLNL while pursuing his Ph.D. from the University of California, Davis. Dr. Crawfis has over 100 publications in the fields of Computer Graphics and Scientific Visualization. His is currently an Associate Professor at The Ohio State University.

Tamara Munzner

Tamara Munzner has been an assistant professor in the University of British Columbia Department of Computer Science since 2002. Her current research interests are information visualization, graph drawing, and dimensionality reduction. She was a research scientist from 2000 to 2002 at the Compaq Systems Research Center in California, and earned her PhD from Stanford between 1995 and 2000. She was on the technical staff of The Geometry Center, a mathematical visualization research group at the University of Minnesota, from 1991 to 1995. Tamara was the IEEE Symposium on Information Visualization (InfoVis) Program/Papers Co-Chair in 2003 and 2004.

Valerio Pascucci

Valerio Pascucci is a Project Leader of the Center for Applied Scientific Computing at Lawrence Livermore national Laboratory and Adjunct Professor of Computer Science at UC Davis. Valerio is Associate Editor of the IEEE Transactions on Visualization and computer Graphics. Valerio's research interests include progressive multi-resolution techniques in scientific visualization, topological analysis, geometric compression, computer graphics, computational geometry, geometric programming, and solid modelling (see www.pascucci.org).

Rita Borgo

Rita Borgo is a post-doctoral research fellow in the Visualization and Virtual Reality Group at the School of Computing, University of Leeds. She received her PhD in Computer Science in 2004 from the University of Pisa in collaboration with the Visual Computing Lab. at the Italian National Research Council in Pisa. Her research interests include volume visualization, computational geometry, 3D image analysis ans synthesis, hierarchical meshes and progressive algorithms, semantic web. More recently her research has focused on using formal techniques for modelling large scale visualization systems and the exploitation of functional languages in the algorithmic development of visualization techniques.

David Duke

David Duke is a Reader in Visualization at the School of Computing of the University of Leeds. His interests in mathematics for computing are wide-ranging, dating back to his 1991 PhD at the University of Queensland on the formal semantics of object-oriented specification. After moving to the UK in 1992 he worked for eight years using formal techniques for modelling interactive and cognitive systems; during this time he also contributed to the development of multimedia standard, again using mathematical specification to explore aspects of the standard. Since 1999 he has worked in visualization, developing tools for graph visualization and, more recently, exploring the use of recent advances in functional programming for large-scale data visualization. He co-chaired the Joint 2005 IEEE/Eurographics Symposium on Data Visualization, and since 2000 has been editor-in-chief of Computer Graphics Forum.

ACKNOWLEDGEMENTS

Rita Borgo and David Duke acknowledge support from EPSRC (A Lazy Polytypic Grid Grant).

REFERENCES

[1] D. C. Banks, S. A. Linton, and P. K. Stockmeyer. Counting cases in substitope algorithms. *IEEE Transactions on Visualization and Computer Graphics*, 10(4):371–384, 2004.

[2] P. Bhaniramka, R. Wenger, and R. Crawfis. Isosurface construction in any dimension using convex hulls. *IEEE Transactions on Visualization and Computer Graphics*, 10(2):130–141, 2004.

[3] H. Carr, B. Duffy, and B. Denby. On histograms and isosurface statistics. *IEEE Transactions on Visualization and Computer Graphics*, 12(5):1259–1266, 2006.

[4] A. J. Hanson, T. Munzner, and G. Francis. Interactive methods for visualizable geometry. *Computer*, 27(7):73–83, 1994.

[5] D. Laney, A. Mascarenhas, P. Miller, P. T. Bremer, and V. Pascucci. Understanding the structure of the turbulent mixing layer in hydrodynamic instabilities. *IEEE Transactions on Visualization and Computer Graphics*, 12(5):1053–1060, 2006.

[6] W. Lorensen and H. Cline. Marching cubes: A high resolution 3d surface construction algorithm. In *Proceedings of SIGGRAPH'87*, pages 163–169. ACM Press, 1987.

[7] T. Munzner. Exploring large graphs in 3d hyperbolic space. *IEEE Comput. Graph. Appl.*, 18(4):18–23, 1998.